Assignment

LEVEL I

1. The differential equation of all circles of radius *a* is of order
(a) 2 (b) 3 (c) 4 (d) none of these

2. The differential equation of all parabolas whose axes are parallel to y-axis is
(a) $\frac{d^3y}{dx^3} = 0$ (b) $\frac{d^2y}{dx^2} = C$ (c) $\frac{d^3y}{dx^3} + \frac{d^2x}{dy^2} = 0$ (d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = C$

3. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is
(a) 5 (b) 4 (c) 3 (d) 2

4. Equation of the curve in which the subnormal is twice the square of the ordinate is given by
(a) $\log y = 2x + \log C$ (b) $y = Ce^{2x}$ (c) $\log y = 2x - \log C$ (d) all are correct

5. The solution of the equation $(x^2 - yx^2)\,dy + (y^2 + x^2y^2)\,dx = 0$
(a) $\frac{1}{x} + y + \log y = C$ (b) $\frac{1}{x} + y + \frac{1}{yz} = C$ (c) $\frac{1}{x} + \frac{1}{y} + \log y - x = C$ (d) none of these

6. The family of curves represented by
$\frac{dy}{dx} = \frac{x^2+x+1}{y^2+y+1}$ and the family represented by $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$
(a) touch each other (b) are orthogonal (c) are one and the same (d) none of these

7. Equation of the curve through the point (1, 0) which satisfy the differential equation $(1 + y^2)\,dx - xy\,dy = 0$ is
(a) $x^2 + y^2 = 1$ (b) $x^2 - y^2 = 1$ (c) $2x^2 + y^2 = 2$ (d) none of these

8. The solution of primitive integral equation $(x^2 + y^2)\,dy = xy\,dx$, is $y = y(x)$., If $y(1) = 1$ and $y(x_0) = e$, then x_0 is
(a) $\sqrt{2(e^2 - 1)}$ (b) $\sqrt{2(e^2 + 1)}$ (c) $\sqrt{3}e$ (d) $\sqrt{\frac{e^2+1}{2}}$

9. The solution of the differential equation $y' = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$ is
(a) $x\,\phi(y/x) = k$ (b) $\phi(y/x) = kx$ (c) $y\,\phi(y/x) = k$ (d) $\phi(y/x) = ky$

10. Solution of the differential equation $(x + y)(dx - dy) = dx + dy$ is
(a) $x + y = ke^{x+y}$ (b) $x - y = ke^{x-y}$ (c) $x + y = ke^{x-y}$ (d) $x - y = ke^{x+y}$

11. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$ then the solution of the equation is
(a) $\log\frac{x}{y} = Cy$ (b) $\log\frac{y}{x} = Cy$ (c) $\log\frac{x}{y} = Cx$ (d) $\log\frac{y}{x} = Cx$

12. The solution of the equation $dy/dx = \cos(x - y)$ is
(a) $y + \cot\left(\frac{x-y}{2}\right) = C$ (b) $x + \cot\left(\frac{x-y}{2}\right) = C$ (c) $x + \tan\left(\frac{x-y}{2}\right) = C$ (d) none of these

13. The degree of the differential equation
$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$ is
(a) 1 (b) 2 (c) 3 (d) none of these

14. The degree of the differential equation satisfying
$\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$ is
(a) 1 (b) 2 (c) 3 (d) none of these

15. The degree of the differential equation
$\left(\frac{d^3y}{dx^3}\right)^{2/3}+4-3\frac{d^2y}{dx^2}+5\frac{dy}{dx}=0$ is
(a) 1 (b) 2 (c) 3 (d) none of these

16. The degree of the differential equation corresponding to the family of curves $y = a(x+a)^2$, where a is an arbitrary constant is
(a) 1 (b) 2 (c) 3 (d) none of these

17. The order and degree of the differential equation of all tangent lines to the parabola $x^2 = 4y$ is
(a) 1, 2 (b) 2, 2 (c) 3, 1 (d) 4, 1

18. Solution of the differential equation x dy – y dx = 0 represents
(a) a parabola whose vertex is at origin (b) a circle whose centre is at origin
(c) a rectangular hyperbola (d) straight lines passing through the origins

19. The equation of the curve in which the portion of the tangent included between the coordinate axes is bisected at the point of contact is
(a) a parabola (b) an ellipse (c) a circle (d) a hyperbola

20. The solution of the differential equation
$\frac{dy}{dx}=\frac{x^2+y^2+1}{2xy}$, satisfying y(1) = 1 is
(a) hyperbola (b) a circle (c) $y^2 = x(1+x) - 10$ (d) $(x-2)^2+(y-3)^2 = 5$

21. The differential equation of all circles which pass through the origin and whose centre lies on y – axis is
(a) $(x^2-y^2)\frac{dy}{dx}-2xy=0$ (b) $(x^2-y^2)\frac{dy}{dx}+2xy=0$
(c) $(x^2-y^2)\frac{dy}{dx}-xy=0$ (d) $(x^2-y^2)\frac{dy}{dx}+xy=0$

22. A curve passes through the point (0, 1) and the gradient at (x , y) on it is y(xy – 1). The equation of the curve is
(a) y(x – 1) = 1 (b) y(x+1) = 1 (c) x(y+1) = 1 (d) x(y – 1) = 1

23. If y(t) is a solution of $(1+t)\frac{dy}{dt}-ty=1$ and y(0) = -1 , then y(1) is equal to
(a) $-\frac{1}{2}$ (b) $e+\frac{1}{2}$ (c) $e-\frac{1}{2}$ (d) $\frac{1}{2}$

24. Let $f:\left[\frac{1}{2},1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. The number the value of $\int_{1/2}^{1} f(x)\,dx$ lies in the interval

(a) $(2e - 1, 2e)$ (b) $(e - 1, 2e - 1)$ (c) $\left(\frac{e-1}{2}, e-1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

25. The solution to the differential equation $ydx + \left(2\sqrt{xy} - x\right)dy = 0$ is

(a) $\ln|y| + \sqrt{\frac{x}{y}} = c$ (b) $\ln|xy| + \sqrt{x} = c\sqrt{y}$ (c) $\sqrt{xy} + x + y = c$ (d) none of these

26. The solution of the differential equation $\frac{dy}{dx} = \frac{1-3y-3x}{1+x+y}$ is

(a) $x + y - \ln |x+y| = c$ (b) $3x + y + 2\ln|1 - x - y| = c$
(c) $x + 3y - 2\ln|1 - x - y| = c$ (d) none of these

27. The expression satisfying the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = 1$ is

(a) $x^2y - xy^2 = c$ (b) $(y^2 - 1)x = y + c$ (c) $(x^2 - 1)y = x + c$ (d) none of these

28. The general solution of the differential equation $(x + 2y^3)\frac{dy}{dx} = y$ is

(a) $x - y^3 = c$ (b) $y = x(x^2 + c)$ (c) $y - x^3 = c$ (d) $x = y(y^2 + c)$

29. The differential equation $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^3 \cos^2 y$ represents a family of curves given by the equation

(a) $x^6 + 6x^2 = C \tan y$ (b) $6x^2 \tan y = x^6 + C$
(c) $\sin 2y = x^3\cos^2 y + C$ (d) none of these

30. Solution of $x^3 \frac{dy}{dx} + 4x^2 \tan y = e^x \sec y$ satisfying $y(1) = 0$ is

(a) $\tan y = 2(x - 2) e^x \log x$ (b) $\sin y = e^x (x - 1) x^{-4}$
(c) $\tan y = 2(x - 2) e^x \log x$ (d) $\sin y = e^x (x - 1) x^{-3}$

31. If y(x) satisfies the differential equation $y' - y \tan x = 2x \sec x$ and $y(0) = 0$, then

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$ (C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

32. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(a) variable radii and a fixed centre at (0, 1)
(b) variable radii and a fixed centre at (0, – 1)
(c) fixed radius 1 and variable centres along the x-axis.
(d) fixed radius 1 and variable centres along the y-axis.

33. If $f(x)$, $g(x)$ be twice differentiable functions on [0, 2] satisfying $f''(x) = g''(x)$, $f'(1) = 2g'(1) = 4$ and $f(2) = 3g(2) = 9$, then $f(x) - g(x)$ at $x = 4$ equals

(a) 0 (b) 10 (c) 8 (d) 2

34. Solution of the differential equation

$$2y\sin x\frac{dy}{dx} = 2\sin x\cos x - y^2\cos x;\ x=\frac{\pi}{2},\quad y=1 \text{ is given by}$$

(a) $y^2 = \sin x$ (b) $y=\sin^2 x$ (c) $y^2=\cos x+1$ (d) none of the

35. The solution of $y\,dx - x\,dy + 3x^2y^2e^{x^3}\,dx = 0$ is

(a) $\frac{x}{y}+e^{x^3}=0$ (b) $\frac{x}{y}-e^{x^3}=0$ (c) $-\frac{x}{y}+e^{x^3}=0$ (d) none of these

36. For the primitive integral equation $ydx+y^2dy;\ x\in R,\ y>0,\ y=y(x),\ y(1)=1$, then $y(-3)$ is
(a) 3 (`b) 2 (c) 1 (d) 5

37. Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1) = 1$. If the y-intercept of the tangent at any point P(x, y) on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to
(a) 3 (b) 6 (c) 9 (d) 12

38. Let f(x) be differentiable on the interval $(0,\infty)$ such that $f(1) = 1$, and $\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1$ for each $x > 0$. Then, f(x) is

(a) $\frac{1}{3x}+\frac{2x^2}{3}$ (b) $-\frac{1}{3x}+\frac{4x^2}{3}$ (c) $-\frac{1}{x}+\frac{2}{x^2}$ (d) $\frac{1}{x}$

39. Solution of the differential equation

$\text{Cos } x\,dy = y(\sin x - y)\,dx,\ 0 < x < \frac{\pi}{2}$, is

(a) $\sec x=(\tan x+c)y$ (b) $y\sec x=\tan x+c$
(c) $y\tan x=\sec x+c$ (d) $\tan x=(\sec x+c)y$

40. The differential equation which represents the family of curves $y = c_1e^{C_2x}$, where c_1 and c_2 are arbitrary constants is
(a) $y'=y^2$ (b) $y''=y'y$ (c) $yy''=y'$ (d) $yy''=(y')^2$

41. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a
(a) ellipse (b) parabola (c) circle (d) hyperbola

42. If $x\frac{dy}{dx}=y(\log y-\log x+1)$, then the solution of the equation is

(a) $\log\left(\frac{x}{y}\right)=cy$ (b) $\log\left(\frac{y}{x}\right)=cx$ (c) $x\log\left(\frac{y}{x}\right)=cy$ (d) $y\log\left(\frac{x}{y}\right)=cx$

43. The order of the differential equation whose general solution is
$y=c_1\cos 2x+c_2\cos^2 x+c_3\sin^2 x+c_4$ is
(a) 2 (b) 4 (c) 3 (d) None of these

44. If the solution of the differential equation $\frac{dy}{dx}=\frac{1}{x\cos y+\sin^2 y}$ is $x = ce^{\sin y}-k(1+\sin y)$, then $k=$

(a) 1 (b) 2 (c) 3 (d) 4