Continuity.

①

Q1 $f(0) = \lim_{x\to 0} \dfrac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{x}$ (f(x) is continuous at $x=0$)

$= \lim_{x\to 0} \dfrac{\dfrac{1\times\cos x}{2\sqrt{1+\sin x}} - \dfrac{(-\cos x)}{2\sqrt{1+\sin x}}}{1} = \dfrac{\frac{1}{2}+\frac{1}{2}}{1} = 1$

Alternative- rationalize the numerator

$f(0) = \lim_{x\to 0} \left[\dfrac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{x}\right] \times \left[\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right]$

$= \lim_{x\to 0} \dfrac{\cancel{1} + \sin x - (\cancel{1} - \sin x)}{x} \cdot \left[\dfrac{1}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right]$

$= 2 \times \dfrac{1}{\left[\sqrt{1+0} + \sqrt{1-0}\right]} = 1$

Q2 $f(0) = \lim_{x\to 0} (x+1)^{\cot x}$ (1^∞ form)

$= e^{\lim_{x\to 0} (x+1-1)\cot x}$

$= e^{\lim_{x\to 0} \frac{x}{\tan x}} = e$

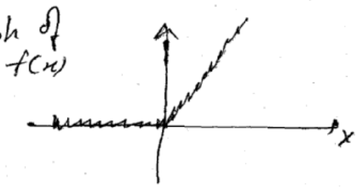
Q3 $f(x) = \begin{cases} \dfrac{x-(-x)}{x}, & x<0 \\ 0, & x = 0 \\ \dfrac{x-x}{x}, & x>0 \end{cases} = \begin{cases} 2, & x<0 \\ 0, & x=0 \\ 0, & x>0 \end{cases}$

clearly f(x) is discontinuous at $x=0$

besides $x=0$, f(x) is continuous at all other values of x.

Q4 $f(x) = \begin{cases} 0, & x<0 \\ 2x, & x\geq 0 \end{cases}$ f(x) is continuous at all x but not differentiable at $x=0$

Graph of $y = f(x)$



(5) $f(x) = [\tan^2 x]$

at $x = 0$ $\underline{L.H.L.}$ $\lim_{x \to 0^-} [\tan^2 x] = 0$

~~At some what less than~~ when x is smaller than zero but near to zero $\tan^2 x$ is small the number so $[\tan^2 x] = 0$

Similarly $\underline{R.H.L.}$ $\lim_{x \to 0^+} [\tan^2 x] = 0$

$f(0) = 0$

Hence $f(x)$ is continuous at $x = 0$

(6) $f(x) = [x] + [x + \frac{1}{3}] + [x + \frac{2}{3}]$

$[x]$ is discontinuous at every integer while $[x+\frac{1}{3}]$ & $[x+\frac{2}{3}]$ are continuous at integer so there addition is discontinous at integer.

$[x+\frac{1}{3}]$ is discontinous at $I - \frac{1}{3}$ or $I + \frac{2}{3}$ while other two are continous so $f(x)$ is discont. at $I - \frac{1}{3}$

Similarly $[x+\frac{2}{3}]$ is discont. at $I + \frac{1}{3}$ or $I - \frac{2}{3}$ so $f(x)$ is ~~co~~ discontinuous at these values of x.

Hence $f(x)$ is discontinuous at $x = \frac{n}{3}$, $n \in I$

(c) $\int_0^{2/3} [x]\,dx + \int_0^{2/3} [x+\frac{1}{3}]\,dx + \int_0^{2/3} [x+\frac{2}{3}]\,dx$

$0 + 0 + \int_0^{1/3} 0 \cdot dx + \int_{1/3}^{2/3} 1 \cdot dx = \frac{1}{3}$

(d) $\lim_{x \to 2/3^+} [x] + [x+\frac{1}{3}] + [x+\frac{2}{3}]$

$0 + 1 + 1 = 2$

$\lim_{x \to 2/3^-} [x] + [x+\frac{1}{3}] + [x+\frac{2}{3}]$

$0 + 0 + 1 = 1$

$L.H.L. \neq R.H.L. \Rightarrow$ limit does not exist.

(2)

(7) Doubt point $\Rightarrow x = 0$ ($1/x$ tends towards ∞)

L.H.L $\lim_{x \to 0^-} \left(\frac{1 - k^{1/x}}{1 + k^{1/x}}\right)$ (when $x \to 0^- \Rightarrow \frac{1}{x} \to -\infty$)

If $k > 1 \Rightarrow \lim_{x \to 0^-} k^{1/x} = k^{-\infty} = 0$

so $\lim_{x \to 0^-} \left(\frac{1 - k^{1/x}}{1 + k^{1/x}}\right) = \frac{1 - 0}{1 + 0} = 1$

R.H.L when $x \to 0^+ \Rightarrow \frac{1}{x} \to \infty$, $k^{1/x} \to \infty$

$$\lim_{x \to 0} \frac{1 - k^{1/x}}{1 + k^{1/x}} = \lim_{x \to 0} \frac{\frac{1}{k^{1/x}} - 1}{\frac{1}{k^{1/x}} + 1} = \frac{0 - 1}{0 + 1} = -1$$

Jump is difference of L.H.L & R.H.L

so jump $= 2$

(8) For continuity at $x = 1$

$\Rightarrow \frac{1}{a} = a \Rightarrow a = \pm 1$

For continuity at $x = \sqrt{2}$

$\Rightarrow a = \frac{2b^2 - 4b}{(\sqrt{2})^2} = b^2 - 2b$

If $a = 1 \Rightarrow b^2 - 2b = 1 \Rightarrow b^2 - 2b - 1 = 0$

$b = \frac{2 \pm \sqrt{4 + 4}}{2} = 1 \pm \sqrt{2}$

If $a = -1 \Rightarrow b^2 - 2b = -1 \Rightarrow b = 1$

(9)
$$\lim_{x \to 0} \frac{(4^x - 1)^3}{(x/p)\underbrace{\frac{\sin x/p}{x/p}}_{1}\underbrace{\frac{\ln(1 + x^2/3)}{x^2/3}}_{1}(x^2/3)} = \lim_{x \to 0} \frac{(4^x - 1)^3}{\frac{x^3}{3p}}$$

$$= \lim_{x \to 0} 3p\left(\frac{4^x - 1}{x}\right)^3$$

$$= 3p(\ln 4)^3$$

For continuity $3p(\ln 4)^3 = 12(\ln 4)^3$

$\Rightarrow p = 4$

(10)

$$f(x) = \begin{cases} \dfrac{x^2-1}{x^2+2(x-1)-1}, & x<1 \\ \frac{1}{2}, & x=1 \\ \dfrac{x^2-1}{x^2-2(x-1)-1}, & x>1 \end{cases} = \begin{cases} \dfrac{x^2-1}{x^2+2x-3}, & x<1 \\ \frac{1}{2}, & x=1 \\ \dfrac{x^2-1}{(x-1)^2}, & x>1 \end{cases}$$

L.H.L. $\lim_{x\to 1} \dfrac{x^2-1}{(x^2+2x-3)} = \lim_{x\to 1} \dfrac{(x-1)(x+1)}{(x+3)(x-1)}$

$= \frac{2}{4} = \frac{1}{2}$

R.H.L. $\lim_{x\to 1^+} \dfrac{x^2-1}{(x-1)^2} = \lim_{x\to 1} \dfrac{x+1}{x-1} = \infty$

L.H.L. $\neq$ R.H.L. $\Rightarrow$ $f(x)$ is discont. at $x=0$

(11)

$$f(x) = \begin{cases} 1, & x \in \text{rational no.} \\ 0, & x \in \text{irrational no.} \end{cases}$$

let $x = a \in$ rational no.

~~L.H.L $\lim_{x\to a} f(x)$~~ ~~If a is rational no. then~~

In a number line near to a rational points there are rational & irrational values.

$\lim_{x\to a} f(x)$ will fluctuate between 1 & 0

so $\lim_{x\to a} f(x)$ does not tends towards a fixed quantity $\Rightarrow$ $\lim_{x\to a} f(x)$ does not exist.

so $f(x)$ is discontinous at any rational value of x.

Similarly $f(x)$ is discontinuous at all the irrational values ~~of~~

Hence $f(x)$ is discontinuous every where.

(12) $f(\theta) = k = \lim_{x\to\theta} \dfrac{\cos x - \cos\theta}{\sin(x-\theta)}$ (Applying L.H. rule)

$= \lim_{x\to\theta} \dfrac{-\sin x}{\cos(x-\theta)} = \dfrac{-\sin\theta}{\cos 0}$

$= -\sin\theta$

(3)

(13) $\lim_{h\to 0} \dfrac{-f'(1-h)-0}{3h^2+3}$ (L-H· rule)

$\lim_{h\to 0} -\dfrac{f'(1)}{3} = -\dfrac{\left[30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x\right]\Big|_{x=1}}{3}$

$= \dfrac{-\left[30 - 56 + 30 - 63 + 6\right]}{3} = \dfrac{53}{3}$

(14) (a) $\tan x$ is discont· at $x = \pi/2$

(b) $f(x) = \int_0^x t \sin\left(\frac{1}{t}\right) dt \Rightarrow f'(x) = x\sin 1/x$ is defined $x \in (0, \pi)$

$\Rightarrow f'(x)$ exist $\Rightarrow f(x)$ is continuous

(c) At $x = 3\pi/4$

L·H·L $\lim_{x\to 3\pi/4^-} f(x) = 1$

R·H·L $\lim_{x\to 3\pi/4^+} 2\sin\left(\frac{2}{a}x\right) = 2\sin\left(\frac{2\times 3\pi}{4a}\right) = 2\sin\frac{3\pi}{2a}$

but $2\sin\frac{3\pi}{2a} \neq 1$, $\forall a \in R \Rightarrow$ discont· at $x = 3\pi/4$

(d) Discontinuous at $x = \pi/2$

(15) $\lim_{x\to 0} \dfrac{x(1+a\cos x) - b\sin x}{x^3} = 1$

$\left(\frac{0}{0}\right)$ form Applying L-H· rule

$\lim_{x\to 0} \dfrac{1 + a\cos x - ax\sin x - b\cos x}{3x^2}$ $\left(\dfrac{1+a-b}{0} \text{ form}\right)$

Since limit is 1 so

$1 + a - b = 0$ —(1)

Again applying L-H· rule $\Rightarrow \lim_{x\to 0} \dfrac{0 - a\sin x - a\sin x - ax\cos x + b\sin x}{6x}$

$\left(\frac{0}{0}\right)$ form again L·H· rule $\Rightarrow \lim_{x\to 0} \dfrac{0 - 2a\cos x - a\cos x + ax\sin x + b\cos x}{6}$

$= \dfrac{0 - 3a + b}{6} = 1 \Rightarrow 0 - 3a + b = 6$

$\Rightarrow -3a + b = 6$ —(2)

From (1) & (2) $a = -5/2$ & $b = -3/2$

(16)
$$f(x) = \begin{cases} |-x|, & -1<x<0 \\ |0|, & 0\le x<1 \\ |x|, & 1\le x<2 \\ 4, & x=2 \end{cases} = \begin{cases} -x, & -1<x<0 \\ 0, & 0\le x<1 \\ x, & 1\le x<2 \\ 4, & x=2 \end{cases}$$

$f(x)$ is discont. at $x = 1, 2$ & cont. but not differentiable at $x = 0$

(17) L.H.L. at $x = 0$

$\lim_{x\to 0^-} (\cos x - \sin x)^{\operatorname{cosec} x}$ (1^∞ form)

$$= e^{\lim_{x\to 0^-} (\cos x - \sin x - 1)\operatorname{cosec} x} = e^{\lim_{x\to 0} \frac{\cos x - \sin x - 1}{\sin x}}$$

$$= e^{\lim_{x\to 0} \frac{-\sin x - \cos x}{\cos x}} = e^{-1}$$

R.H.L. $\lim_{x\to 0^+} \dfrac{e^{1/x} + e^{2/x} + e^{3/x}}{ae^{2/x} + be^{3/x}}$ as $x \to 0^+ \Rightarrow \frac{1}{x} \to \infty$, $e^{1/x} \to \infty$

dividing both numerator & denominator by $e^{3/x}$

$$\Rightarrow \lim_{x\to 0^+} \frac{\frac{e^{1/x}}{e^{3/x}} + \frac{e^{2/x}}{e^{3/x}} + \frac{e^{3/x}}{e^{3/x}}}{\frac{ae^{2/x}}{e^{3/x}} + b\frac{e^{3/x}}{e^{3/x}}} = \lim_{x\to 0^+} \frac{\frac{1}{e^{2/x}} + \frac{1}{e^{1/x}} + 1}{\frac{a}{e^{1/x}} + b}$$

$$= \frac{0+0+1}{0+b} = \frac{1}{b}$$

For continuity.

$e^{-1} = a = \frac{1}{b} \Rightarrow a = 1/e,\ b = e$

(18) $f(x+y) = f(x)\, f(y)$

differentiating w.r.to x keeping y constant.

$f'(x+y) = f'(x)\, f(y)$

sub. $x = 0 \Rightarrow f'(y) = f'(0)\, f(y) = 2f(y)$

replacing $y \to x \Rightarrow f'(x) = 2f(x)$

$$\Rightarrow \frac{f'(x)}{f(x)} = 2$$

$$\Rightarrow \int \frac{f'(x)}{f(x)}\,dx = \int 2\,dx + c$$

$$\Rightarrow \ln f(x) = 2x + c \Rightarrow f(x) = e^{2x+c} = e^c e^{2x}$$

$f(x) = e^{c} e^{2x} = C_1 e^{2x}$ (4)

Since $f(x+y) = f(x) f(y)$

$y = 0 \Rightarrow f(x) = f(x) f(0) \Rightarrow f(0) = 1 \quad (f(x) \neq 0)$

so $f(0) = 1$

$f(x) = C_1 e^{2x}$

$f(0) = C_1 e^{0} \Rightarrow C_1 = f(0) = 1$

$\Rightarrow f(x) = 1 \cdot e^{2x} = e^{2x}$ (continuous & differentiable $\forall x \in R$.)

(19) R.H.L. $\lim_{x\to 0+} x\left(\dfrac{e^{[x]+|x|}-2}{[x]+|x|}\right)$

$= \lim_{x\to 0+} x\left(\dfrac{e^{0+x}-2}{0+x}\right) = \lim_{x\to 0+} (e^{x}-2) = -1$

L.H.L. $\lim_{x\to 0-} x\left(\dfrac{e^{[x]+|x|}-2}{[x]+|x|}\right) = \lim_{x\to 0-} x\left(\dfrac{e^{-1-x}-2}{-1-x}\right)$

$= 0\left(\dfrac{e^{-1}-2}{-1}\right) = 0$

(20) $f(x) = \frac{1}{2}x - 1$

$\dfrac{1}{f(x)} = \dfrac{1}{\frac{x}{2}-1}$ is not defined when $\frac{x}{2} - 1 = 0$

$\Rightarrow x = 2$

$\Rightarrow \dfrac{1}{f(x)}$ is discontinuous at $x = 2$

$$\tan\left[\tfrac{x}{2}-1\right] = \begin{cases} \tan(-1), & 0 \le x < 2 \\ \tan 0, & 2 \le x \le \pi \end{cases}$$

$\parallel$

$\tan\left(\left[\tfrac{x}{2}\right]-1\right)$

$\Rightarrow \tan[x/2 - 1]$ is discontinuous at $x = 2$

(21) at $x = \pi/4$

L.H.L. $\lim_{x\to \pi/4-} x + a\sqrt{2}\sin x = \pi/4 + a\sqrt{2}\cdot\frac{1}{\sqrt{2}} = \pi/4 + a$

R.H.L. $\lim_{x\to \pi/4+} 2x\cot x + b = \pi/2 + b$

For cont. $\pi/4 + a = \pi/2 + b \Rightarrow a - b = \pi/4$ —(1)

Similarly at $x = \pi/2$

L.H.L. $= 2\cdot\pi/2 \cot \pi/2 + b = b$ —(2)

R.H.L. $= a\cos\pi - b\sin\pi/2 = -a - b$

For continuity $b = -a - b$

$\Rightarrow 2b + a = 0$ — (2)

Find a & b from (1) & (2)

(22) $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^{2}$

$\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right) = 1$ & $\lim_{x \to \infty} 2x = \infty$

$\Rightarrow \; = e^{\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2} - 1\right) \times 2x}$

$= e^{\lim_{x \to \infty} \left(2a + \frac{2b}{x}\right)} = e^{2a}$

So $e^{2a} = e^{2} \Rightarrow$ $\underline{a = 1 \; \& \; b \in R.}$

(23) In number line a rational point is surrounded by many rational & irrational points.

If $f(x)$ is continous & $f(2) = 10$ & not constant then it will take the irrational values too. But $f(x)$ takes only rational values $\Rightarrow f(x)$ is constant function

so $f(x) = 10, \; \forall \; \underline{x \in R}$

(24) Doubt points are integer.

let $x = a \in I$

$f(a) = [a] \cos\left\{\left(\frac{2a-1}{2}\right)\pi\right\}$

$= a \times 0$

$= 0$

since angle of cosine is $(2a-1)\pi/2$

$(\text{odd integer}) \cdot \pi/2$

cosine of odd integral multible of $\pi/2 = 0$

$\Rightarrow \cos\left\{(2a-1)\pi/2\right\} = 0$

R.H.L. $\lim_{x \to a^+} [x] \cos\left(\frac{2x-1}{2}\right)\pi$

$= a \cos\left(\frac{2a-1}{2}\right)\pi$

$= 0$

L.H.L. $\lim_{x \to a^-} [x] \cos\left(\frac{2x-1}{2}\right)\pi$

$(a-1) \cos\left(\frac{2a-1}{2}\right)\pi$

$= 0$

$\Rightarrow f(x)$ is cont. at integer $x = a \in I$

(25) $f(x)$ is not defined when $\log|\sin x| = 0$, $|\sin x| = 0$

$\Rightarrow |\sin x| = 1$, $|\sin x| = 0$

$\Rightarrow x = (2n+1)\pi/2$, $x = n\pi$

$f(x)$ is not defined at infinite no. of points so $f(x)$ is discontinuous at infinite points.

(26)

$$x = \begin{cases} 3t, & t \le 0 \\ t, & t \ge 0 \end{cases}, \quad y = \begin{cases} 0, & t \le 0 \\ 2t^2, & t \ge 0 \end{cases}$$

$$y = \begin{cases} 0, & x/3 \le 0 \\ 2x^2, & x \ge 0 \end{cases}$$

$$y = \begin{cases} 0, & x \le 0 \\ 2x^2, & x \ge 0 \end{cases}$$

at $x = 0$, y is continuous & differentiable

$\Rightarrow y$ is cont. & diff $\forall\ x \in R$.

(27)

$$K = \lim_{x \to 0} \frac{[9^x(4^x - 1) - 1(4^x - 1)][\sqrt{2} + \sqrt{1+\cos x}]}{[\sqrt{2} - \sqrt{1+\cos x}][\sqrt{2} + \sqrt{1+\cos x}]}$$

$$= \lim_{x \to 0} \frac{(9^x - 1)(4^x - 1)[\sqrt{2} + \sqrt{2}]}{[2 - (1 + \cos x)]}$$

$$= \lim_{x \to 0} \frac{(9^x - 1)(4^x - 1)(2\sqrt{2})}{2\dfrac{\sin^2 x/2}{(x/2)^2} \cdot (x/2)^2}$$

$$= \lim_{x \to 0} 4 \cdot (2\sqrt{2}) \frac{(9^x - 1)}{x} \frac{(4^x - 1)}{x}$$

$$= 8\sqrt{2}(\ln 9)\ln 4$$

$$= 16\sqrt{2}(\ln 3)(\ln 2)$$

(28)

$$R = \lim_{x\to\frac{\pi}{2}} \frac{1-\cos(\pi/2-x)}{[\pi-2x]^2}\cdot\frac{\ln \sin x}{\ln[1+\pi^2-4\pi x+4x^2]}$$

$$= \lim_{x\to\pi/2} \underbrace{\frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{16\left(\frac{\pi}{4}-\frac{x}{2}\right)^2}}_{=1}\cdot\frac{\ln \sin x}{\underbrace{\ln[1+(\pi-2x)^2]}_{}}\cdot\underbrace{\frac{(\pi-2x)^2}{(\pi-2x)^2}}_{=1}$$

$$= \lim_{x\to\pi/2} \frac{1}{8}\,\frac{\ln \sin x}{[\pi-2x]^2}$$

$$= \lim_{x\to\pi/2} \frac{1}{8}\cdot\frac{\frac{1}{\sin x}\cos x}{-4(\pi-2x)} \qquad (\text{L.H. rule})$$

$$= \lim_{x\to\pi/2} -\frac{1}{8}\underbrace{\frac{\sin(\pi/2-x)}{(\sin x)(4)(2)(\pi/2-x)}}_{=1}$$

$$= -\frac{1}{8}\times\frac{1}{8} = -\frac{1}{64}$$

(29) Only doubt pt. is $x=0$. $f(x)$ is given to be cont. in $[-1,1]$ $\Rightarrow$ cont. at $x=0$

L.H.L. $$\lim_{x\to 0-}\frac{\sqrt{1+px}-\sqrt{1-px}}{x} = \lim_{x\to 0}\frac{\frac{p}{2\sqrt{1+px}}+\frac{p}{2\sqrt{1-px}}}{1}$$

$$= p$$

R.H.L. $$\lim_{x\to 0+}\frac{2x+1}{x-2} = -\frac{1}{2}$$

From cont. $p = -\frac{1}{2}$

(31) L.H.L. $$\lim_{x\to 0-} a\sin\frac{\pi}{2}(x+1) = a\sin\frac{\pi}{2} = a$$

R.H.L. $$\lim_{x\to 0+}\frac{\tan x-\sin x}{x^3} = \lim_{x\to 0}\frac{\sin x\left(\frac{1}{\cos x}-1\right)}{x\ \ x^2}$$

$$= \lim_{x\to 0}\underbrace{\left(\frac{\sin x}{x}\right)}_{=1}\left(\frac{1-\cos x}{x^2\cos x}\right) = \lim_{x\to 0}\frac{2\sin^2 x/2}{(\cos x)\frac{x^2}{2^2}\times 2^2}$$

$$= \frac{2}{4} = \frac{1}{2}$$

For cont. $a = \frac{1}{2}$

6

(32) $\cancel{f(x) = \begin{cases} -\ln|x|, & 0<|x|<1 \\ \ln|x|, & |x|>1 \end{cases}} = \begin{cases} -\ln(-x \\ \end{cases}$

(32) Graph of f(x) is symmetric about y axis

for +ive values of x

$$f(x) = \begin{cases} -\ln x, & 0<x\le 1 \\ \ln x, & x>1 \end{cases}$$

f(x) is cont. at $x=1$, L.H.L = R.H.L. = 0

$f'(1-) = -\frac{1}{x}\Big|_{x=1} = -1$ = L.H.D

$f'(1+) = \frac{1}{x}\Big|_{x=1} = 1$ = R.H.D.

but L.H.D. ≠ R.H.D. ⇒ non diff. at $x=1$

Since f(x) is symmetric about y axis so f(x) is non diff. at $x=-1$ also.

(33) $F(x) = |\cos x|$

Doubt points are odd multiple of $\pi/2$

$$f(x) = \begin{cases} -\cos x & -3\pi/2 \le x \le -\pi/2 \\ \cos x, & -\pi/2 \le x \le \pi/2 \\ -\cos x, & \pi/2 \le x \le 3\pi/2 \\ \cos x, & 3\pi/2 \le x \le 5\pi/2 \end{cases}$$

f(x) is continuous at $x = \pi/2, 3\pi/2, 5\pi/2$ - - —

L.H.L. = R.H.L. = 0

$f'(\pi/2+) = -(-\sin x)\Big|_{x=\pi/2} = 1$

$f'(\pi/2-) = -\sin x\Big|_{x=\pi/2} = -1$

So f(x) is not differentiable at $x = \pi/2$

Similarly at $x = \pm 3\pi/2, \pm 5\pi/2$, - - —

So f(x) is continuous but not differentiable at $x = (2n+1)\pi/2$

(34) For cont. at $x=1$

L.H.L. = R.H.L

$1+3+a = b+2$ — (1)

For differentiable at $x=1$

$f'(1-) = f'(1+)$

$2+3 = b$ — (2) ⇒ $b=5$ — (2)

From ① & ② $\Rightarrow b = 5$ & $a = 4$

(35) (i) $\lim_{x\to 1} \dfrac{\int_1^{x^2}[f(t)-t]\,dt}{(x-1)^2}$ ($\frac{0}{0}$ form)

$\lim_{x\to 1} \dfrac{[f(x^2)-x^2]\times 2x}{2(x-1)}$ (L.H. rule)

$\lim_{x\to 1} \dfrac{[f(x^2)-x^2]}{2(x-1)} \times 2\times 1$ ($\frac{0}{0}$ form)

$\lim_{x\to 1} \dfrac{\cancel{2}[f'(x^2)\times 2x - 2x]}{\cancel{2}}$ (L.H. rule)

$2f'(1) - 2 = 6-2 = 4$

(ii) $\lim_{n\to\infty} \left(\dfrac{1+4^{1/n}}{2}\right)^n$, $\lim_{n\to\infty} \dfrac{1+4^{1/n}}{2} = 1$

$= e^{\lim_{n\to\infty}\left(\frac{1+4^{1/n}}{2}-1\right)\cdot n}$

$= e^{\lim_{n\to\infty} \frac{4^{1/n}-1}{2\cdot 1/n}} = e^{\frac{1}{2}\ln 4} = e^{\ln 2} = 2$

(iii) $\lim_{n\to\infty} \dfrac{2x}{\pi}\tan^{-1}(nx)$, $x>0$ $\Rightarrow nx \to \infty$ as $n\to\infty$

$= \dfrac{2x}{\pi}\times \pi/2$

$= x$

$\lim_{x\to 0^+} [x-1] = -1$

(IV) $\lim_{n\to\infty}\left[\sum_{r=1}^{n} \dfrac{1}{2^r}\right] = \lim_{n\to\infty}\left(\dfrac{\frac{1}{2}(1-(1/2)^n)}{1-1/2}\right)$

$= \lim_{n\to\infty}[1-(1/2)^n]$

$1-(1/2)^n$ is smaller than one so

$\lim_{n\to\infty}[1-(1/2)^n] = 0$