LIMITS
MULTIPLE CHOICE QUESTIONS

LEVEL 1 (Objective Questions)

1. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ equals

(a) $-\pi$ (b) π (c) $\frac{\pi}{2}$ (d) 1

2. For $x \in R$, $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ equals

(a) e (b) e^{-1} (c) e^{-5} (d) e^5

3. $\lim_{h\to 0} \frac{\ln(1+2h) - 2\ln(1+h)}{h^2}$ equals

(a) –1 (b) 0 (c) 1 (d) does not exist

4. $\lim_{n\to\infty}\left(\frac{1^3 + 2^3 + \ldots\ldots + n^3}{n^4}\right)$ is equal to

(a) 0 (b) 1/4 (c) 1/2 (d) 1/8

5. $\lim_{x\to -\infty}\left(\frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{1 + |x|^3}\right)$ equals

(a) 0 (b) 1 (c) ∞ (d) –1

6. The limiting value of $(\cos x)^{\frac{1}{\sin x}}$ as $x \to 0$ is

(a) 1 (b) e (c) 0 (d) none of these

7. The value of $\lim_{x\to 0}\left(\frac{2^x - 1}{\sqrt{1+x} - 1}\right)$ is

(a) $2\ln 2$ (b) $\ln 2$ (c) $(\ln 2)/2$ (d) none of these

8. $\lim_{x\to -\pi} \frac{|x+\pi|}{\sin x}$

(a) 1 (b) –1 (c) 0 (d) limit does not exist

9. Let $a_{n+1} = \sqrt{2 + a_n}$, n = 1, 2, 3, ….. and $a_1 = 3$. Then $\lim_{n\to\infty} a_n$ is

(a) –1 (b) 2 (c) $\sqrt{5}$ (d) 3

10. The value of $\lim_{x\to 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is

(a) $\frac{1}{8\sqrt{3}}$ (b) $\frac{1}{4\sqrt{3}}$ (c) 0 (d) none of these

11. $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$ equals

(a) $\frac{\pi}{2}$ (b) $e^{\frac{\pi}{2}}$ (c) $e^{-\frac{\pi}{2}}$ (d) 1

12. The value of $\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{\frac{1}{x}}$ is

(a) e (b) e^2 (c) e^3 (d) none of these

13. $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{\sin ax-\sin bx}$ $(a \neq b)$ equals

(a) −1 (b) 0 (c) 1/2 (d) 1

14. $\lim_{x\to 0}\frac{\log_{10}\left(1+4x+3x^2+2x^3\right)}{\left(1+2x+3x^2+4x^3\right)}$ equals

(a) $\log_e 10$ (b) $\log_{10} e$ (c) 1 (d) 0

15. Given that $f(a) = 2$, $g(a) = -1$, $f'(a) = 1$, $g'(a) = 2$, the value of $\lim_{x\to a}\frac{f(x)g(a)-f(a)g(x)}{x-a}$ is

(a) 5 (b) −5 (c) 1/5 (d) −1/5

16. $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) none of these

17. $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$ is

(a) $\pi/2$ (b) $\pi + 2$ (c) $2/\pi$ (d) 2π

18. $\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ is

(a) 0 (b) 1/2 (c) log 2 (d) none of these

19. If $f'(2) = 2$, $f''(2) = 1$, then $\lim_{x\to 2}\frac{2x^2-4f'(x)}{x-2}$ is

(a) 4 (b) 0 (c) 2 (d) ∞

20. $\lim_{x\to 0}\frac{1}{x}\left(\int_y^a e^{\sin^2 t}\,dt-\int_{x+y}^a e^{\sin^2 t}\,dt\right)$ is equal to (where a is a constant)

(a) $e^{\sin^2 y}$ (b) $\sin 2y\, e^{\sin^2 y}$ (c) 0 (d) none of these

21. Let f: R → R be a differentiable function such that $f(2) = 2$. Then the value of

$\lim_{x\to 2}\int_{2}^{f(x)}\frac{4t^3}{x-2}\,dt$ is

(a) $6f'(2)$ (b) $12f'(2)$ (c) $32f'(2)$ (d) none of these

22. If a, b, c, d are positive, then $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx} =$

(a) $e^{d/b}$ (b) $e^{c/a}$ (c) $e^{(c+d)/a+b}$ (d) e

23. $\lim_{x\to 0}\frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ is

(a) 2 (b) −2 (c) ½ (d) −1/2

24. Let f: R → R be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals

(a) 1 (b) $e^{1/2}$ (c) e^2 (d) e^3

25. If $\lim_{x\to 0}\frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to

(a) 0 (b) (n + 1)/n (c) n (d) n + 1/n

26. $\lim_{x\to 0}\frac{\sin x^4 - x^4\cos x^4 + x^{20}}{x^4(e^{2x^4} - 1 - 2x^4)}$ is equal to

(a) 0 (b) $-\frac{1}{6}$ (c) $\frac{1}{6}$ (d) does not exist

27. $\underset{x\to 3}{\text{Limit}}\frac{(x^3+27)\ln(x-2)}{(x^2-9)} =$

(a) −8 (b) 8 (c) 9 (d) −9

28. $\underset{x\to 0^+}{\text{Limit}}\frac{\cos^{-1}(1-x)}{\sqrt{x}} =$

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 1 (d) 0

29. The value of $\underset{x\to 0}{\text{Limit}}\frac{\sin(\ell n(1+x))}{\ell n(1+\sin x)}$ is:

(a) 0 (b) 1/2 (c) 1/4 (d) 1

30. The value of $\underset{x\to\pi/2}{\text{Limit}}\tan^2 x\left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2}\right)$ is equal to:

(a) 1/10 (b) 1/11 (c) 1/12 (d) 1/8

31. $\lim_{n\to\infty}\frac{-3n+(-1)^n}{4n-(-1)^n}$ is

(a) $-\frac{3}{4}$ (b) $-\frac{3}{4}$ if n is even: $\frac{3}{4}$ if n is odd

(c) not exist if n is even: $-\frac{3}{4}$ if n is odd (d) +1 if n is even; does not exist if n is odd

32. $\underset{n\to\infty}{\text{Limit}}\ n\cos\left(\frac{\pi}{4n}\right)\sin\left(\frac{\pi}{4n}\right)$ has the value equal to:

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) none

33. The limit $\lim_{x\to a}\left(2-\frac{a}{x}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ is equal to

(a) $e^{-a/\pi}$ (b) $e^{-2a/\pi}$ (c) $e^{-2/\pi}$ (d) 1

34. $\underset{n\to\infty}{\text{Limit}}\ \frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}}=$

(a) 5 (b) 3 (c) 1 (d) zero

35. $\underset{x\to\pi/2}{\text{Limit}}\ \frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$ is

(a) $\frac{1}{16}$ (b) $-\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $-\frac{1}{32}$

36. $\underset{x\to0^+}{\text{Lim}}\ \log_{\sin(x/2)}\sin x$ is equal to

(a) 1 (b) 0 (c) 4 (d) $\frac{1}{4}$

LEVEL - II

37. If $f(x)=\begin{cases}\sin x, & x\neq n\pi,\ n=0,\pm1,\pm2,\pm3,\ldots\ldots\\ 2, & \text{otherwise}\end{cases}$ and $g(x)=\begin{cases}x^2+1, & x\neq0,2\\ 4, & x=0\\ 5, & x=2\end{cases}$

then $\lim_{x\to0}g(f(x))$ equals

(a) 4 (b) 5 (c) 1 (d) does not exist

38. If [x] denotes the greatest integer $\le x$, then the value of $\lim_{x\to1}\{1-x+[x-1]+[1-x]\}$ is

(a) 0 (b) 1 (c) −1 (d) none of these

39. If $f(x)=\begin{cases}x^2, & x\in Z\ \text{(set of integers)}\\ \frac{k(x^2-4)}{2-x}, & x\notin Z\end{cases}$

then $\lim_{x\to2}f(x)$ exists when

(a) $k=1$ (b) $k\in R$ (c) $x\in R-\{1\}$ (d) does not exist

40. The value of $\lim_{x\to1}\left[\sin^{-1}x\right]$ is ([.] denotes the greatest integer function)

(a) does not exist (b) 1 (c) 0 (d) $\frac{\pi}{2}$

41. $\lim_{x\to 0} \frac{ae^{\frac{1}{x}} + be^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}$, where $a \neq b$ equals

(a) 1 (b) a (c) b (d) does not exist

42. $\lim_{n\to\infty}\left(\frac{1}{1.3} + \frac{1}{3.5} + \text{.........} + \text{upto n terms}\right)$ is

(a) 1/4 (b) 1/2 (c) 1 (d) 2

43. $\lim_{x\to 0}\frac{\cos(\sin x) - \cos x}{x^4}$ is

(a) 1/5 (b) 1/4 (c) 1/2 (d) none of these

44. $\lim_{x\to\infty}\frac{x^n}{e^x} = 0$, (n integer), for

(a) no value of n
(b) all values of n
(c) only negative values of n
(d) only positive values of n

45. The value of $\lim_{x\to 0^+} x^m (\log x)^n$, $m, n \in N$ is

(a) 0 (b) m/n (c) m n (d) none of these

46. The value of $\lim_{x\to\infty}\frac{\log x}{x^n}$, $n > 0$ is

(a) 0 (b) 1 (c) 1/n (d) 1/n!

47. Let $f(x) = \lim_{n\to\infty}\frac{x^{2n} - 1}{x^{2n} + 1}$, then

(a) $f(x) = 1$, for $|x| > 1$
(b) $f(x) = -1$ for $|x| < 1$
(c) $f(x)$ is not defined for any value of x
(d) $f(x) = 1$ for $|x| = 1$

48. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2\, dt}{x \sin x}$ is

(a) 3/2 (b) 1 (c) –1 (d) none of these

49. If $f(x) = \frac{\sin[x]}{[x]}$, $[x] \neq 0$

$= 0$, $[x] = 0$

where [x] denotes the greatest integer less than or equal to *x*, then $\lim_{x\to 0} f(x)$ equals:

(a) 1 (b) 0 (c) –1 (d) none of these

50. The integer n for which $\lim_{x\to 0}\frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non – zero number is

(a) 1 (b) 2 (c) 3 (d) 4

51. $\lim_{x\to 0}\frac{x\cos x - \sin x}{x^2 \sin x} =$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

52. $\lim_{h\to 0}\left[\dfrac{1}{h\sqrt[3]{8+h}}-\dfrac{1}{2h}\right]=$

(a) $\dfrac{1}{12}$ (b) $-\dfrac{4}{3}$ (c) $-\dfrac{16}{3}$ (d) $-\dfrac{1}{48}$

53. If $f(n+1)=\dfrac{1}{2}\left\{f(n)+\dfrac{9}{f(n)}\right\}$ $n\in N$ and $f(n)>0$ for all $n\in N$ then $\lim\limits_{n\to\infty} f(n)$ is equal to

(a) 3 (b) -3 (c) 1 /2 (d) none of these

54. The value of $\lim\limits_{x\to\infty}\int_0^x\left(\dfrac{1}{\sqrt{1+t^2}}-\dfrac{1}{1+t}\right)dt$ is

(a) 2 (b) 0 (c) ln 2 (d) e^2

55. If α and β are the root of the quadratic equation $ax^2+bx+c=0$, then $\lim\limits_{x\to\frac{1}{\alpha}}\sqrt{\dfrac{1-\cos(cx^2+bx+a)}{2(1-\alpha x)^2}}=$

(a) $\left|\dfrac{c}{2\alpha}\left(\dfrac{1}{\alpha}-\dfrac{1}{\beta}\right)\right|$ (b) $\left|\dfrac{c}{2\beta}\left(\dfrac{1}{\alpha}-\dfrac{1}{\beta}\right)\right|$ (c) $\left|\dfrac{c}{\alpha\beta}\left(\dfrac{1}{\alpha}-\dfrac{1}{\beta}\right)\right|$ (d) None of these

56. $\underset{x\to 0}{\text{Lt}}\left[\dfrac{\sin[x-3]}{[x-3]}\right]$ where [.] denotes greatest integer functions is

(a) 0 (b) 1 (c) does not exist (d) sin 1

57. If $\underset{x\to 0}{\text{Lim}}\dfrac{x^3}{\sqrt{a+x}(bx-\sin x)}=1$ then the constants 'a' and 'b' are (where $a>0$)

(a) $b=1, a=36$ (b) $a=1, b=6$ (c) $a=1, b=36$ (d) $b=1, a=6$

58. $\underset{x\to 0}{\text{Lim}}\dfrac{\sin([x^2])}{x^2}$ where [.] denote the greatest integer function.

(a) is 1 (b) is 0 (c) does not exist (d) None of these

59. $\underset{x\to 0}{\text{Limit}}\dfrac{e^{-x^2/2}-\cos x}{x^3\sin x}=$

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{12}$ (d) $\dfrac{1}{8}$

60. If $f(x)=\begin{cases}x-1, & x\ge 1\\ 2x^2-2, & x<1\end{cases}$, $g(x)\begin{cases}x+1, & x>0\\ -x^2+1, & x\le 0\end{cases}$ and $h(x)=|x|$

then find $\underset{x\to 0}{\text{Lim}}\, f(g(h(x)))$

(a)1 (b)0 (c)-1 (d)does not exists

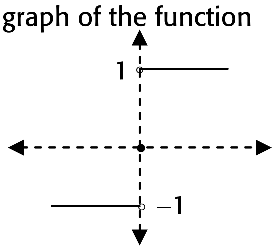
61. Let α,β be the roots of $ax^2+bx+c=0$, where $1<\alpha<\beta$. Then $\underset{x\to x_0}{\text{Lim}}\dfrac{|ax^2+bx+c|}{ax^2+bx+c}=1$ then which of the following statements is incorrect

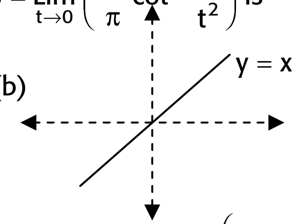
(a) $a>0$ and $x_0<1$ (b) $a>0$ and $x_0>\beta$
(c) $a<0$ and $\alpha<x_0<\beta$ (d) $a<0$ and $x_0<1$

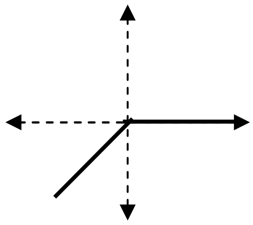
62. The limit $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$ is equal to

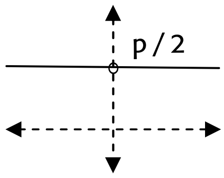
(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 1

63. The graph of the function $f(x)=\lim_{t\to 0}\left(\frac{2x}{\pi}\cot^{-1}\frac{x}{t^2}\right)$ is

(a)


(b)


(c)


(d)


64. If $\lim_{x\to\infty} f(x)$ exist and is finite & nonzero and if $\lim_{x\to\infty}\left(f(x)+\frac{3f(x)-1}{f^2(x)}\right)=3$ then the value of $\lim_{x\to\infty} f(x)$ is

(a) 1 (b) -1 (c) 2 (d) none of these

65. The value of $\lim_{n\to\infty} n^2\left(\sqrt[n]{a}-\sqrt[n+1]{a}\right), a>0$ is

(a) $\ell n a$ (b) e^a (c) e^{-a} (d) none of these

66. If $\lim_{x\to 0}[1+x\ell n(1+b^2)]^{1/x}=2b\sin^2\theta,\ b>0$ and $\theta\in(-\pi,\pi)$, then the value of θ is

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

67. The value of $\lim_{x\to 0}\left((\sin x)^{1/x}+(1+x)^{\sin x}\right)$, where $x>0$ is

(a) 0 (b) – 1 (c) 1 (d) 2

68. $\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2-\frac{\pi^2}{16}}$ equal

(a) $\frac{8}{\pi}f(2)$ (b) $\frac{2}{\pi}f(2)$ (c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (d) $4f(2)$

69. Let $f: R\to R$ be a positive increasing function with $\lim_{x\to 0}\frac{f(3x)}{f(x)}=1$

Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 1

70. Let f be a real-valued function defined on the interval (– 1, 1) such that $e^{-x}f(x)=2+\int_0^x\sqrt{t^4+1}\,dt$, for all $x\in(-1,1)$, and let f^{-1} be the inverse function of f. Then $(f^{-1})(2)$ is equal to

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

More than one

66. Let $f(x)=\dfrac{\cos 2-\cos 2x}{x^2-|x|}$, then

(a) $\lim\limits_{x\to -1} f(x)=2\sin 2$ (b) $\lim\limits_{x\to 1} f(x)=2\sin 2$ (c) $\lim\limits_{x\to -1} f(x)=2\cos 2$ (d) $\lim\limits_{x\to 1} f(x)=2\cos 2$

67. Let $f(x)=\dfrac{\sqrt{x^2+2}}{3x-6}$

(a) $\lim\limits_{x\to -\infty} f(x)=-\dfrac{1}{3}$ (b) $\lim\limits_{x\to \infty} f(x)=\dfrac{1}{3}$ (c) $\lim\limits_{x\to -\infty} f(x)=\dfrac{1}{3}$ (d) $\lim\limits_{x\to \infty} f(x)=-\dfrac{1}{3}$

68. If $\text{Limit}_{x\to 0}(\cos x+a\sin bx)^{1/x}=e^2$, then the possible value of 'a' & 'b' is

(a) a =1 b = 2 (b) a = 2, b = 1 (c) a = 3 , b = 2/3 (d) a = 2/3 , b = 3

69. If $\ell=\lim\limits_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=\lim\limits_{x\to 0}\dfrac{1+a\cos x}{x^2}-\lim\limits_{x\to 0}\dfrac{b\sin x}{x^3}$, where $\ell\in R$, then

(a) $(a,b)=(-1,0)$ (b) a & b are any real numbers (c) $\ell=0$ (d) $\ell=\dfrac{1}{2}$

70. If $\text{Limit}_{x\to 0}\dfrac{\sin 2x+a\sin x}{x^3}=p$ (finite), then

(a) $a=-2$ (b) $a=-1$ (c) $p=-2$ (d) $p=-1$

71. $\lim\limits_{x\to\infty}\dfrac{(ax+1)^n}{x^n+A}=$

(a) a^n if $n\in N$ (b) ∞ if $n\in Z^-$ & $a=A=0$ (c) $\dfrac{1}{1+A}$ if $n=0$ (d) a^n if $n\in Z^-$, $A=0$ & $a\neq 0$

72. Let $L=\lim\limits_{x\to 0}\dfrac{a-\sqrt{a^2-x^2}-\dfrac{x^2}{4}}{x^4}$, $a>0$

If L is finite, then

(a) $a=2$ (b) $a=1$ (c) $L=\dfrac{1}{64}$ (d) $L=\dfrac{1}{32}$

Answers : Limits

LEVEL – I(Objective Questions)

1	b	8	d	15	b	22	a	29	d	36	a
2	c	9	b	16	a	23	c	30	c		
3	a	10	a	17	c	24	c	31	a		
4	b	11	d	18	b	25	d	32	b		
5	d	12	b	19	a	26	c	33	c		
6	a	13	d	20	a	27	c	34	d		
7	a	14	d	21	c	28	b	35	c		

LEVEL – II & More than One

37	c	44	B	51	d	58	b	65	a	72	a,b
38	c	45	a	52	d	59	c	66	d		
39	b	46	a	53	a	60	b	67	c		
40	a	47	a, b	54	c	61	d	68	a		
41	d	48	b	55	a	62	c	69	d		
42	b	49	d	56	c	63	c	70	b		
43	D	50	c	57	a	64	a	71	ab		