Math Solutions

TYRP-14 (JEE Main) (1)

Q61 $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3 = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ (a)

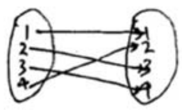
$= -(a+b+c)\left[\frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\}\right] < 0$

Q62 $\sin\left(\sin^{-1}\frac{1}{\sqrt{1+(1+x)^2}}\right) = \cos\cos^{-1}\frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}} \Rightarrow 1+(1+x)^2 = 1+x^2$

$\Rightarrow 1+x = \pm x$

$\Rightarrow x = -\frac{1}{2}$

Q63  $\Rightarrow$ One to One & Onto.

Q65 $y = \frac{x^2-x}{x^2+2x} \Rightarrow (y-1)x^2 + (2y+1)x = 0$

$\Rightarrow x = 0$ or $x = \frac{2y+1}{1-y}$

but $x \neq 0 \Rightarrow x = \frac{2y+1}{1-y} = f^{-1}(y)$

$f^{-1}(x) = \frac{2x+1}{1-x}$

$\frac{d}{dx} f^{-1}(x) = \frac{3}{(1-x)^2}$

Q66 $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{vmatrix} = 0 = 1(3a-25) - 2(a-10) + 3(5-6) = 0$

$\Rightarrow 3a - 25 - 2a + 20 - 3 = 0$

$\Rightarrow a - 8 = 0 \Rightarrow a = 8$

If $a = 8$ then Eqn. can be written as.

$x_1 + 2x_2 + 3x_3 = 6$ — (1)

$x_1 + 3x_2 + 5x_3 = 9$ — (2)

$2x_1 + 5x_2 + 8x_3 = b$ — (3)

(1) + (2) $\Rightarrow 2x_1 + 5x_2 + 8x_3 = 15$

so if Eqn. have infinite sol. $\Rightarrow b = 15$

(67) Relation is reflexive & transitive but not symmetric (2)

(68) $\tan^{-1}\frac{x}{\sqrt{1-x^2}} = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \frac{2x}{1-x^2},\ x \neq \pm 1 \Rightarrow$ either $x = 0$

or

$\frac{1}{\sqrt{1-x^2}} = \frac{2}{1-x^2}$

$\Rightarrow 1 = \frac{2}{\sqrt{1-x^2}}$

$\Rightarrow 1-x^2 = 4 \Rightarrow$ not possible.

but $x = \pm 1$ satisfies the Eqn. so Eq. have three solutions

(69)
$3p + 3q + 2r = 3$ — (1)
$4p + 2q + 0\cdot r = 0$ — (2)
$p + 3q + 2r = 1$ — (3)

(1) − (3) $\Rightarrow 2p = 2 \Rightarrow p = 1$

from (2) $\Rightarrow 4 + 2q = 0 \Rightarrow q = -2$

From (1) $\Rightarrow$ ~~$6 - 12 + 2r = 3$~~ $\Rightarrow 1 - 6 + 2r = 1$
$\Rightarrow r = 3$

(70) System have nontrivial solution

$$\Rightarrow \begin{vmatrix} 1 & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$ & $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & \sin\alpha & \cos\alpha \\ 0 & \cos\alpha - \sin\alpha & \sin\alpha - \cos\alpha \\ 0 & -2\sin\alpha & 0 \end{vmatrix} = 0 \Rightarrow 2\sin\alpha(\sin\alpha - \cos\alpha) = 0$$

$\Rightarrow \sin\alpha = 0$ or
$\sin\alpha = \cos\alpha$

In $(0, \pi/2)$ only one $\alpha = \pi/4$ satisfies it. so statement one is correct.

For statement 2
applying $R_1 \rightarrow R_1 + R_3$

$$\Rightarrow \begin{vmatrix} 2\cos\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0 \Rightarrow 2\cos\alpha(-\cos^2\alpha - \sin^2\alpha) = -2\cos\alpha = 0$$

does not have any sol. in $(0, \pi/2)$

(3)

Q71. ~~$I_r = Tan^{-1}$~~

$$S = \tan^{-1}\frac{1}{1+n(n+1)} + \tan^{-1}\frac{1}{1+(n+1)(n+2)} + \tan^{-1}\frac{1}{1+(n+2)(n+3)} + \text{— —} \tan^{-1}\frac{1}{1+(n+19)(n+20)}$$

$$S = \left[\tan^{-1}(n+1) - \tan^{-1}n\right] + \left[\tan^{-1}(n+2) - \tan^{-1}(n+1)\right] + \left[\tan^{-1}(n+3) - \tan^{-1}(n+2)\right] + \text{— —} \left[\tan^{-1}(n+20) - \tan^{-1}(n+19)\right]$$

$$= -\tan^{-1}n + \tan^{-1}(n+20) = \tan^{-1}\frac{20}{1+n(n+20)}$$

Q72 let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^2 = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow a^2+bc=1, \quad bc+d^2=1, \quad b(a+d)=0, \quad c(a+d)=0$

$b(a+d)=0, \quad c(a+d)=0$

$\Rightarrow a+d=0$ or b & $c=0$

if b & $c=0 \Rightarrow a$ & $d=\pm 1$ non possible

so $a+d=0$

$Tr(A)=0 \Rightarrow$ statement 1 is correct.

$\det A = ad-bc = -(a^2+bc) = -1$

$= a(-a)-bc \Rightarrow$ statement 2 is correct.

but not correct explanation.

(73) $x^2-4xy+3y^2=0 \Rightarrow x^2-xy-3xy+3y^2=0$

$\Rightarrow (x-y)(x-3y)=0 \Rightarrow x=y$ or $x=3y$

$(x,x)\in R, \forall x\in N \Rightarrow$ Reflexive relation

$(x,y)\in R \Rightarrow x=3y \Rightarrow (3y,y)\in R$ but $(y,3y)\notin R$

$\Rightarrow$ not symmetric.

$(3y,y)\in R, (y,y/3)\in R$ but $(3y, y/3)\notin R \Rightarrow$ not transitive.

(4)

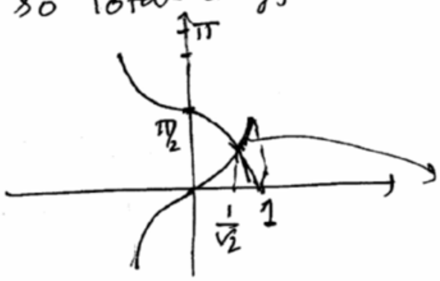
(74) Unique sol. ⇒ trivial sol. so
$\Delta \neq 0$

(75) $S = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{11} \end{bmatrix} = a_{11}^2 - a_{21}a_{12} \neq 0$

a_{11}	a_{12}	a_{21}	
0	(1,2)	1, 2	→ 4 ways.
1	≠1	≠1	→ 8 ways (all possible pair of a_{12} & a_{21} except (1,1))
2	≠2	≠2	→ 8 ways

so Total ways 20

(76)



portion of $\sin^{-1}x$ where $\sin^{-1}x > \cos^{-1}x$
so $x \in \left(\frac{1}{\sqrt{2}}, 1\right]$

(77) $2y = x + z$

$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$\Rightarrow \tan^{-1}\frac{2y}{1-y^2} = \tan^{-1}\frac{x+z}{1-xz}$

$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xz} = \frac{2y}{1-xz}$

$\Rightarrow y^2 = xz \Rightarrow x, y, z$ are in G.P.

x, y, z are also in A.P. $\Rightarrow x = y = z$.

(78) $|P| = |\text{adj } A| = 12 - 12 - \alpha(4-6) + 3(4-6)$

$= 2\alpha - 6$

$|A| = 4 \Rightarrow \det(\text{adj } A) = 4^2$

$2\alpha - 6 = 16 \Rightarrow \alpha = 11$

(79) $\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$

$\Rightarrow (k+1)(k+3) = 8k \Rightarrow k^2 - 4k + 3 = 0 \Rightarrow k = 1, 3$

(5)

but $k=1$ does not satisfy $\frac{8}{k+3} \neq \frac{4k}{3k-1}$

so $k=3$ only.

(82) $AB = I$

(83) $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3$

$\begin{vmatrix} (\alpha+2) & (\alpha+2) & (\alpha+2) \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0.$

$= (\alpha+2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

$C_2 \to C_2 - C_1$
$C_3 \to C_3 - C_1$

$\Rightarrow (\alpha+2) \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha-1 & 0 \\ 1 & 0 & \alpha-1 \end{vmatrix} = 0$

$\Rightarrow \alpha = -2, 1$

when $\alpha = 1$, system is homogeneous $\Rightarrow$ infinite sol.

$\alpha = -2$ system is following.

$\Rightarrow$ $-2x + y + z = -3$
$x - 2y + z = -3$
$x + y - 2z = -3$

$\Delta_x = \begin{vmatrix} -3 & 1 & 1 \\ -3 & -2 & 1 \\ -3 & 1 & -2 \end{vmatrix} = -3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = (-3) \begin{vmatrix} 1 & 0 & 0 \\ 1 & -3 & 0 \\ 1 & 0 & -3 \end{vmatrix}$

$= -27 \neq 0$

$\Rightarrow \alpha \neq 2$ so only one value of α that is 1

(84) ~~$R_1 \to R_1 + R_2 + R_3$~~ $C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} 2x + (a^2+b^2+c^2)x & (1+b^2)x & (1+c^2)x \\ 2x + (a^2+b^2+c^2)x & 1+b^2x & (1+c^2)x \\ 2x + (a^2+b^2+c^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$

$\begin{vmatrix} 0 & \checkmark & \checkmark \\ 0 & \checkmark & \checkmark \\ 0 & \checkmark & \checkmark \end{vmatrix} = 0$ because $a^2+b^2+c^2 = -2$

(86) $-1 \le \frac{x}{2} - 1 \le 1 \Rightarrow 0 \le x \le 4$ — (1) (6).
$\ln(\cos x)$ is defined for $\forall\, x \in (-\pi/2, \pi/2)$ — (2)
so common of (1) & (2) is $[0, \pi/2)$

(88) $f : (-1, 1) \to B$
$f(x) = \tan^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$
f is onto ⇒ Range = co-domain
$-1 < x < 1 \Rightarrow -\pi/4 < \tan^{-1}x < \pi/4$ so Range $= (-\pi/2, \pi/2)$

(89) $-1 \le x - 3 \le 1 \Rightarrow 2 \le x \le 4$ — (1) & $9 - x^2 > 0$
$\Rightarrow -3 < x < 3$ — (2)

common of (1) & (2) $2 \le x < 3$

(90) Every odd natural no. have it's image +ve integers & even natural no. have it's image −ve integer so function is both onto & one one.