MATHEMATICS

SECTION A

(*Each Question carries 1 Mark*)

1. $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of x + y.

2. Find $\vec{a}.\left(\vec{b}\times\vec{c}\right)$, if $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}+2\hat{k}$

3. Let R be the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {a, b) : 2 divides (a–b)}. Write the equivalence class [0].

4. Write the Principal value of $\cos^{-1}[\cos(680°)]$

5. If A is an invertible matrix of order 3 and |A| = 5, then find (adj.A |

6. L and M are two points with position vectors $2\vec{a}-\vec{b}$ and $2\vec{a}+\vec{b}$ respectively. Write the position vector of a point N which divides the line segment LM in the ratio 2 : 1 externally.

SECTION B

(*Each Question carries 4 Marks*)

7. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ \frac{-1}{\sqrt{2}} \le x \le 1$

8. Find the particular solution of the differential equation $\frac{dy}{dx} = 1+x+y+xy$, given that y = 0 when x = 1.

9. A line passes through (2, –1, 3) and is perpendicular to the line $\vec{r} = (\hat{i}+\hat{j}+\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r} = (2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$ optain its equation in vector and Cartesian from.

10. An experiment succeeds thrice as often as it fails. Find the probability that in the next k five trials, there will be at least 3 successes.

11. Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z = 0. Also find the distance of the plane, obtained above, from the origin.

12. Using the properties of determinants, prove the following :

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(bc+ca+ab)$$

13. If $(x-y).e^{\frac{x}{x-y}} = a$, prove that $y\frac{dy}{dx}+x = 2y$

14. Find the general solution of the differential equation $(x-y)\frac{dy}{dx} = x+2y$

15. $\vec{a} = 2\hat{i}-3\hat{j}+\hat{k}$, $\hat{b} = -\hat{i}+\hat{k}$, $\vec{c} = 2\hat{j}-\hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a}+\vec{b})$ and $(\vec{b}+\vec{c})$.

16. Show that the relation S in the set R of real numbers, defined as $S = \{(a, b) : a, b \in R \text{ and } a \le b^3\}$ is neither reflexive, nor symmetric nor transitive.

17. Show that $f : N \to N$, given by $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is odd} \end{cases}$ is both one-one and onto.

18. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

19. Prove the following : $\cos\left[\tan^{-1}\left\{\sin\left(\cos^{-1}x\right)\right\}\right] = \sqrt{\frac{1+x^2}{2+x^2}}$

20. Solve for x : $\cos^{-1}x + \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{6}$

21. Using properties of determinants, prove the following :

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

22. Find the value of 'a' for which the function f defined as $f(x) = \begin{cases} a \sin\frac{\pi}{2}(x+1) & , \quad x \le 0 \\ \frac{\tan x - \sin x}{x^3} & , \quad x > 0 \end{cases}$

is continuous at $x = 0$

23. If $x = a \sin t$ and $y = a\left(\cos t + \log \tan\frac{t}{2}\right)$, find $\frac{d^2y}{dx^2}$.

24. Verify that y = 3 cos (log x) + 4 sin (log x), is a solution of the differential equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

25. Evaluate : $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}$

26. Evaluate : $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$

27. Find the area of the region given by $\{(x, y) : x^2 \le y \le |x|\}$

28. Sketch the graph of y = |x + 3| and evaluate the area under the curve y = |x + 3| above x–axis and between x = – 6 to x = 0.

29. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

30. A die thrown again and until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

SECTION C
(Each Question carries 6 Marks)

31. Find the distance of the point (2, 12, 5) from the point of intersection of the line

$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$

32. A bag contains 3 red and 7 black balls. Two balls are selected at random one-by-one without replacement. If the second selected ball happens to be red, what is the probability that the first selected balls also red?

33. Find the vector and cartesian forms of the equation of the plane passing through the point (1, 2, –4) and parallel to the lines $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = \hat{i} - 3\hat{j} + 5\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$. Also, find the distance of the point (9, –8, –10) from the plane thus obtained.

34. A total amount of Rs. 7,000 is deposited in three different saving bank accounts with annual interest rates of 5%, 8% and 8½% respectively. The total annual interest from these three accounts is Rs. 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices.

35. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of equations $x - y = 3$, $2x + 3y + z = 17$ and $y + 2z = 7$.

36. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

37. If the sum of the lengths of the hypotenuse and a side of a right triangle is given, then show that the area of the triangle is maximum, when the angle between them is 60°.

38. Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point (1, 4).

39. Evaluate $\int \frac{1}{\sin^4 x + \sin^2 x \cos^2 + \cos^4 x} dx$.

40. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs. 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

41. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to both clubs. Find the probability of the lost card being of clubs.

SOLUTIONS

2. $\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$

4. $\cos^{-1}\cos(680°) = \cos^{-1}\cos(720 - 40) = \cos^{-1}\cos 40° = 40°$

5. $|\text{Adj } A| = |A|^{n-1}$ where n is order of matrix.

7. Sub $x = \cos\theta \Rightarrow \tan^{-1}\left[\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right] = \tan^{-1}\left[\frac{\sqrt{2}\cos\theta/2 - \sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2 + \sqrt{2}\sin\theta/2}\right]$

$$= \tan^{-1}\left(\frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}\right) = \tan^{-1}\left(\frac{\frac{\cos\theta/2 - \sin\theta/2}{\sin\theta/2}}{\frac{\cos\theta/2 + \sin\theta/2}{\sin\theta/2}}\right) = \tan^{-1}\left(\frac{1-\tan\theta/2}{1+\tan\theta/2}\right) = \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

8. $\frac{dy}{dx} = (1+x)(1+y) \Rightarrow \int \frac{dy}{1+y} = \int (1+x)\,dx$

9. Required line is perpendicular to both the lines, hence it's | | vector is perpendicular to parallel vector of both lines

$$\Rightarrow \vec{b} \perp 2\hat{i} - 2\hat{j} + \hat{k} \text{ and } \hat{i} + 2\hat{j} + \hat{k} \text{ so } \vec{b} = (2\hat{i} - 2\hat{j} + \hat{k}) \times (\hat{i} + 2\hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 6\hat{k}$$

10. Let prob. Of success be p and failure be q, $q = \frac{p}{3}$ so $p + \frac{p}{3} = 1 \Rightarrow p = \frac{3}{4}$ and $n = 5$

$$P(x \geq 3) = P(x=3) + P(x=4) + P(x=5) = {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5$$

11. Let the plane $x + y + z - 1 + \lambda(2x + 3y + 4z - 5) = 0 \Rightarrow (1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z - 1 - 5\lambda = 0$ (1)

is $\perp$ to $x - y + z = 0 \Rightarrow \vec{n}_1 . \vec{n}_2 = 0 \Rightarrow \vec{n}_1 = (1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}$ and $\vec{n}_2 = \hat{i} - \hat{j} + \hat{k}$

$\vec{n}_1 . \vec{n}_2 = 0 = (1+2\lambda) - (1+3\lambda) + 1 + 4\lambda \Rightarrow 3\lambda + 1 = 0 \Rightarrow \lambda = -\frac{1}{3}$ (Substitute λ is (1))

12. $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\begin{vmatrix} a-b & a^2-b^2 & c(b-a) \\ b-c & b^2-c^2 & a(c-b) \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 1 & a+b & -c \\ 1 & b+c & -a \\ c & c^2 & ab \end{vmatrix}$$

Again $R_2 \to R_1 - R_2$ $= (a-b)(b-c)\begin{vmatrix} 0 & a-c & a-c \\ 1 & b+c & -a \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)\begin{vmatrix} 0 & -1 & -1 \\ 1 & b+c & -a \\ c & c^2 & ab \end{vmatrix}$

$c_2 \to c_2 - c_3$

$$(a-b)(b-c)(c-a)\begin{vmatrix} 0 & 0 & -1 \\ 1 & a+b+c & -a \\ c & c^2-ab & ab \end{vmatrix} = (a-b)(b-c)(c-a)\left[-(c^2-ab) + c(a+b+c)\right]$$

$$= (a-b)(b-c)(c-a)[-c^2 + ab + ac + bc + c^2] = (a-b)(b-c)(c-a)[ab + bc + ca]$$

13. Taking log both side

$\ln(x-y)e^{x/x-y} = \ln a \Rightarrow \ln(x-y) + \frac{y}{x-y}\ln e = \ln a$

$$\frac{1 - \frac{dy}{dx}}{x-y} + \frac{-y + x\frac{dy}{dx}}{(x-y)^2} = 0$$

$$(x-y)\left(1-\frac{dy}{dx}\right)-y+x\frac{dy}{dx}=0 \quad x-\frac{x\,dy}{dx}-y+\frac{y\,dy}{dx}-y+\frac{x\,dy}{dx}=0 \Rightarrow y\frac{dy}{dx}+x=2y$$

14. Homogenious differentiation equation, solve by substituting $y/x = t$

15. Area of parallelogram $=\frac{1}{2}\left|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right|$

16. (i) Since $a \le a^3$ is not true for $\forall\ a \in R$

For example $a=\frac{1}{2} \Rightarrow \frac{1}{2} \le \frac{1}{8}$ so (a, a) $\notin$ S for $\forall\ a \in R$ $\Rightarrow$ relation is not reflexive.

(ii) If (a, b) $\in S \Rightarrow a \le b^3$ $a \le b^3$ does not imply that $b \le a^3 \Rightarrow$ (b, a) need not belong to S.

For Ex. $(1, 2) \in S$ but $(2, 1) \notin S \Rightarrow$ not symmetric relation.

(iii) If $(a, b) \in S \Rightarrow a \le b^3$, $(b, c) \in R \Rightarrow b \le c^3$

But from this we can not conclude that $a \le c^3$.

For Ex. $(10, 3) \in S$, $(3, 2) \in S$ but $(10, 2) \notin S \Rightarrow S$ is not transitive.

17. $f(x)=\begin{cases} x+1, & x \in \text{ odd no.} \\ x-1, & x \in \text{ even no.}\end{cases}$

$\Rightarrow$ f(1) = 2, f(2) = 1, f(3) = 2, f(4) = 3 and so on $\Rightarrow$ every odd natural no. have next even no. as it's image and every even no. has previous odd no. as it's image so function is one to one and image includes all the natural no. so f(x) is onto function.

18. Let $\sin^{-1}\frac{3}{4}=\theta$ .... (1) $\Rightarrow$ $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\tan\frac{\theta}{2}$

from (1) $\sin\theta=\frac{3}{4}$ $\Rightarrow$ $\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}=\frac{3}{4} \Rightarrow 3\tan^2\frac{\theta}{2}-8\tan\frac{\theta}{2}+3=0$

$\Rightarrow$ $\tan\frac{\theta}{2}=\frac{-(-8)\pm\sqrt{64-4\times3\times b}}{2\times3}=\frac{8\pm\sqrt{28}}{6}=\frac{4\pm\sqrt{7}}{3}$

Since $\theta=\sin^{-1}\frac{3}{4} \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$ so $\frac{\theta}{2} \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan\frac{\theta}{2} < 1$ so $\tan\frac{\theta}{2}=\frac{4-\sqrt{7}}{3}$

19. $\cos\left[\tan^{-1}\{\sin(\cos^{-1}x)\}\right]=\cos\left[\tan^{-1}\left\{\sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right\}\right] = \cos\left[\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right]$

$=\cos\cos^{-1}\frac{\sqrt{1+x^2}}{\sqrt{1+1+x^2}}=\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$

20. $\cos^{-1}x+\sin^{-1}\frac{x}{2}=\frac{\pi}{6}$ $\Rightarrow$ $\cos^{-1}x=\frac{x}{6}-\sin^{-1}\frac{\pi}{2}$

$x=\cos\left(\frac{\pi}{6}-\sin^{-1}\frac{x}{2}\right)=\cos\frac{\pi}{6}\cos\sin^{-1}x+\sin\frac{\pi}{6}\sin\sin^{-1}x$

$$x=\frac{\sqrt{3}}{2}\sqrt{1-x^2}+\frac{1}{2}.x \Rightarrow \frac{x}{2}=\frac{\sqrt{3}}{2}\sqrt{1-x^2} \Rightarrow \frac{x^2}{4}=\frac{3}{4}(1-x^2) \Rightarrow x=\pm\frac{\sqrt{3}}{2}$$

21. $R_2 \to R_2 - R_1 \Rightarrow \begin{vmatrix} a & b & c \\ -b & -c & -a \\ b+c & c+a & a+b \end{vmatrix}$

$$R_3 \to R_3 + R_2 \Rightarrow \begin{vmatrix} a & b & c \\ -b & -c & -a \\ c & a & b \end{vmatrix} = a[-bc-(-a^2)]-b[-b^2+ac]+c[-ac-(-c^2)] = a^3+b^3+c^3-3abc$$

22. Left hand limit at $x=0$

$$\lim_{h\to 0} f(0-h)=\lim_{h\to 0} a\sin\frac{\pi}{2}(-h+1)=a\sin\frac{\pi}{2}=a$$

R.H.L. $\lim_{h\to 0} f(0+h)=\lim_{h\to 0}\frac{\tan h-\sin h}{h^3}=\lim_{h\to 0}\frac{\frac{\sin h}{\cos h}-\sin h}{h^3}$

$$=\lim_{h\to 0}\frac{\sin h\,(1-\cos h)}{\cos h.\,h^3}=\lim_{h\to 0}\frac{(\sin h)\ (2\sin^2\frac{h}{2})}{\cos h\,.\,h^3}=\lim_{h\to 0}\frac{2}{\cos h}\left(\frac{\sin h}{h}\right).\left(\frac{\sin^2 h/2}{h^2}\right)$$

$$=\lim_{h\to 0}\frac{2}{\cos h}.\left(\frac{\sin h}{h}\right)\left(\frac{\sin h/2}{h/2}\right)^2\times\frac{1}{4}\quad=\frac{2}{1}.1.1\frac{1}{4}=\frac{1}{2}$$

For continuity L.H.L. = R.H.L. $\Rightarrow$ $a = 1/2$

23. $$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{a\left(-\sin t+\frac{\sec^2\frac{t}{2}}{\tan\frac{t}{2}}.\frac{1}{2}\right)}{a\cos t}=\frac{a\left(-\sin t+\frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right)}{\cos t}=\frac{-\sin t+\frac{1}{\sin t}}{\cos t}=\frac{1-\sin^2 t}{\cos t\ \sin t}$$

$$=\frac{\cos^2 t}{\cot\sin t}=\cot t\ ,\ \frac{d^2y}{dx^2}=\frac{d}{dx}(\cot t)=-\text{cosec}^2 t.\frac{dt}{dx}=-\text{cosec}^2 t.\frac{1}{a\cos t}\ =-\frac{1}{a\sin^2 t\cos t}$$

24. $y=3\cos(\ln x)+4\sin(\ln x) \Rightarrow \frac{dy}{dx}=\frac{3\sin(\ln x)}{x}-\frac{4\cos(\ln x)}{x} \Rightarrow x\frac{dy}{dx}=3\sin(\ln x)-4\cos(\ln x)$

Again diff. w.r.t. x $\ x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{3\cos(\ln x)}{x}+\frac{4\sin(\ln x)}{x}$

$$x^2\frac{d^2y}{dx^2}+\frac{x\,dy}{dx}=3\cos(\ln x)+4\sin(\ln x)=y$$

25. $$\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx\ \int\frac{2\cos^2 x-1-(2\cos^2\alpha-1)}{\cos x-\cos\alpha}dx=2\int\frac{\cos^2 x-\cos^2\alpha}{\cos x-\cos\alpha}dx$$

$$=2\int(\cos x+\cos\alpha)\,dx=2\sin x+2(\cos\alpha)\,x+c$$

26. Let $I=\int_0^1 \frac{\ln(1+x^2)}{1+x^2}\,dx$ sub. $x=\tan\theta \Rightarrow \int_0^{\pi/4}\frac{\ln(1+\tan\theta)}{1+\tan^2\theta}\sec^2\theta\, d\theta$

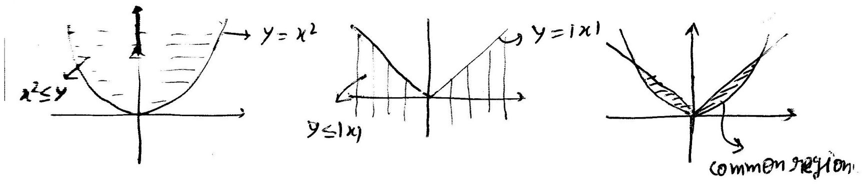
$I=\int_0^{\pi/4}\ln(1+\tan\theta)\,d\theta$ ......(1)

$I=\int_0^{\pi/4}\ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta=\int_0^{\pi/4}\ln\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta=\int_0^{\pi/4}\ln\left(\frac{2}{1+\tan\theta}\right)d\theta$

$I=\int_0^{\pi/4}[\ln 2-\ln(1+\tan\theta)]d\theta=\int_0^{\pi/4}\ln 2\, d\theta-\int_0^{\pi/4}\ln(1+\tan\theta)\,d\theta$

$I=\frac{\pi}{4}\ln 2-I \Rightarrow I=\frac{\pi}{8}\ln 2$

27. $x^2 \le y \le |x| \Rightarrow$ we need common region of $x^2 \le y$ and $y \le |x|$



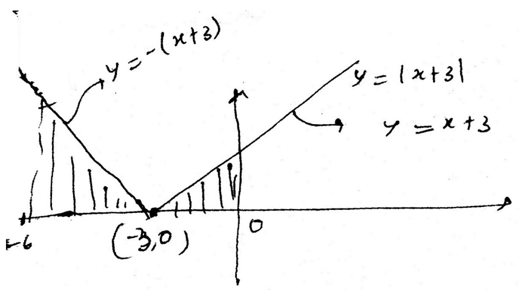
Region is symmetric about y-axis.

Point of intersection of $y = x^2$ and $y = x$ is (1, 1)

So, required area $\Rightarrow 2\int_0^1 (y_{line}-y_{curve})\,dx = 2\int_0^1 (x-x^2)\,dx = 2\left[\frac{1}{2}-\frac{1}{3}\right]=\frac{1}{3}$

28. Required area =

$\int_{-6}^{0}|x+3|\,dx=\int_{-6}^{-3}-(x+3)\,dx+\int_{-3}^{0}(x+3)\,dx$



29. Since $|\bar{a}+\bar{b}|=|a| \Rightarrow |\bar{a}+\bar{b}|^2=|\bar{a}|^2$

$\Rightarrow |a|^2+|b|^2+2\bar{a}.\bar{b}=|a|^2$

$\Rightarrow |b|^2+2\bar{a}.\bar{b}=0 \Rightarrow \bar{b}.\bar{b}+2\bar{a}.\bar{b}=0 \Rightarrow \bar{b}.(\bar{b}+2\bar{a})=0 \Rightarrow \bar{b}\perp \bar{b}+2\bar{a}$

30. Third six in sixth throw of die means in first five throws there are two six and in the sixth throw six should come.

$E_1 \rightarrow$ event in first five throws 2 six and 3 non six occurs

$E_2 \rightarrow$ event in sixth throw six occurs.

$P(E_1 \cap E_2)=P(E_1)\,P(E_2)={}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2\times\frac{1}{6}$

31. For intersection of line and plane, let the point on the line $(2+3\lambda,\ -4+4\lambda, 2+2\lambda)$ and it lies on the plane

$\vec{r}\,.(\hat{i}-2\hat{j}+\hat{k})=0$ or $x-2y+2=0$

$\Rightarrow \quad 2+3\lambda-2(-4+4\lambda)+2+2\lambda=0$

$\Rightarrow \quad -3\lambda+12=0 \Rightarrow \lambda=4$

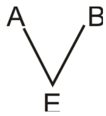
So intersection point is (14, 12, 10)

It's distance from (2, 12, 5) is $=\sqrt{12^2+0^2+5^2}=13$

32. Let A be event first ball drawn is red

B be event first ball drawn is black

E be event second ball drawn is red



By bayes theorem

$$P(A/E)=\frac{P(A)\,P\left(\frac{E}{A}\right)}{P(A)\,P\left(\frac{E}{A}\right)+P(B)\,P\left(\frac{E}{B}\right)}$$

$$=\frac{\frac{3}{10}\times\frac{2}{9}}{\frac{3}{10}\times\frac{2}{9}+\frac{7}{10}\times\frac{3}{9}}=\frac{6}{27}=\frac{2}{9}$$

33. Plane is parallel to given lines $\Rightarrow$ normal to plane is $\perp$ to parallel vectors of both lines

$\Rightarrow \quad \vec{n}\perp 2\hat{i}+3\hat{j}+6\hat{k}$ and $\hat{i}+\hat{j}-\hat{k}$

So Let $n=(2\hat{i}+3\hat{j}+6\hat{k})\times(\hat{i}+\hat{j}-\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ 1 & 1 & -1\end{vmatrix}=-9\hat{i}+8\hat{j}-\hat{k}$$

Plane passes through (1, 2, –4)

So Eqn. of plane is $\vec{r}\,.(-9\hat{i}+8\hat{j}-\hat{k})=(-9\hat{i}+8\hat{j}-\hat{k})\,.(\hat{i}+2\hat{j}-4\hat{k})$

$=\vec{r}\,.(-9\hat{i}+8\hat{j}-\hat{k})=-9i+16+4=11$ or $-9x+8y-z=11$

34. Let Rs. x are deposited in both 5% and 8% interest rate account. Amount in 8.5% account = 7000 – x

Let row matrix represents the amount in three accounts

[x x, 7000 – x] ……(1)

and column matrix represents interest rate/100

So interest matrix $= \begin{bmatrix} 5/100 \\ 8/100 \\ 8.5/100 \end{bmatrix}$ .....(2)

Total interest in a year is multiplication of (1) and (2)

$$\Rightarrow \quad [x \quad x \quad 7000-x]\begin{bmatrix} 5/100 \\ 8/100 \\ 8.5/100 \end{bmatrix} = [550] \quad = \left[\frac{5x}{100} + \frac{8x}{100} + \frac{(7000-2x)\,8.5}{100}\right] = [550]$$

$\Rightarrow \quad x = 1125$ Rs.

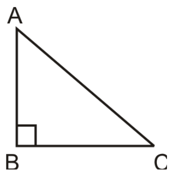
35. $AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$

System of Eqn. in matrix form can be written as $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$AX = C \Rightarrow X = A^{-1}C$

Since $AB = 6\,I \Rightarrow A\left(\frac{B}{6}\right) = I \;\Rightarrow A^{-1} = \frac{B}{6}$ so $x = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

36. Let $AB = x$ and $BC = y$ so $AC = \sqrt{x^2 + y^2} = \text{constant} \;\Rightarrow\; x^2 + y^2 = \text{cosntant} = C$ ....(1)



Area of $\Delta = \frac{1}{2}xy \;= \frac{1}{2}x\sqrt{c - x^2}$ (from Eqn. 1)

When area is maximum, (area)² will also be maximum so $\Delta^2 = \frac{1}{4}x^2\,(c - x^2) = \frac{1}{4}(x^2c - x^4)$

$\frac{d\Delta^2}{dx} = \frac{1}{4}(2xc - 4x^3)$ for Δ^2 to be maximum/min.

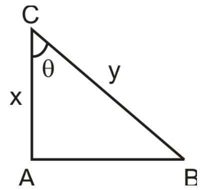
$\frac{d\Delta^2}{dx} = 0 \Rightarrow 2xc - 4x^3 = 0 \Rightarrow x = \sqrt{\frac{c}{2}}$

$$\Rightarrow \quad \frac{d^2\Delta^2}{dx^2} = \frac{1}{4}(2c - 12x^2) \;\Rightarrow\; \left.\frac{d^2\Delta^2}{dx^2}\right|_{x=\sqrt{\frac{c}{2}}} = \frac{1}{4}\left(2c - 12\frac{c}{2}\right) = -\frac{1}{4}(4c) < 0$$

So when $x = \sqrt{\frac{c}{2}}$ Δ^2 is maximum $\Rightarrow$ Δ is maximum

It $x = \sqrt{\frac{c}{2}}$ from (1) $\frac{c}{2} + y^2 = c \Rightarrow y = \sqrt{\frac{c}{2}} \Rightarrow x = y$ triangle is isosceles.

37. Here x + y = constant = c



$AB = \sqrt{y^2 - x^2} = \sqrt{(c-x)^2 - x^2} = \sqrt{c^2 - 2cx}$

Area $(\Delta) = \frac{1}{2} x.\sqrt{c^2 - 2c\,x} \Rightarrow \Delta^2 = \frac{1}{4} x^2(c^2 - 2cx) = \frac{1}{4}(c^2x^2 - 2cx^3)$

$\Rightarrow \frac{d\Delta^2}{dx} = \frac{1}{4}(2c^2x - 6cx^2) \Rightarrow \frac{d\Delta^2}{dx} = 0 \Rightarrow 2c^2x = 6cx^2 \Rightarrow c = 3x$

$\frac{d^2\Delta^2}{dx^2} = \frac{1}{4}(2c^2 - 12cx)$

$\left.\frac{d^2\Delta^2}{dx^2}\right|_{x=\frac{c}{3}} = \frac{1}{4}(2c^2 - 12\,c.\frac{c}{3}) = \frac{1}{4}(-2c^2) < 0$ so when x = c/3 Δ^2 is maximum.

If x = c/3 $\Rightarrow$ y = c − c/3 = 2c/3 so $\cos\theta = \frac{x}{y} = \frac{c/3}{2c/3} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

38. Let the point on the curve $y^2 = 2x$ be (h, k) $\Rightarrow$ $k^2 = 2h \Rightarrow h = k^2/2$ so point is $(k^2/2, k)$

$l = \sqrt{\left(\frac{k^2}{2} - 1\right)^2 + (k-4)^2} \Rightarrow l^2 = \frac{k^4}{4} + 1 - k^2 + k^2 + 16 - 8k = \frac{k^4}{4} - 8k + 17$

$\frac{dl^2}{dk} = k^3 - 8 \Rightarrow \frac{dl^2}{dk} = 0 \Rightarrow k^3 - 8 = 0 \Rightarrow k = 2$

$\frac{d^2l^2}{dk^2} = 3k^2$

$\left.\frac{d^2l^2}{dk^2}\right|_{k=2} = 12 > 0 \Rightarrow$ when k = 2 l^2 is minimum hence l is minimum so point $\left(\frac{4}{2}, 2\right) = (2, 2)$

39. $\int \frac{dx}{\sin^4 x + \cos^4 x + \sin^2 x \cos^2 x}$

$\int \frac{dx}{\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x} = \int \frac{dx}{(\sin^2 x + \cos^2 x)^2 - \sin^2 x \cos^2 x}$

$\int \frac{dx}{1 - \frac{4\sin^2 x \cos^2 x}{4}} = \int \frac{dx}{1 - \frac{1}{4}\sin^2 2x} = \int \frac{4\,dx}{4 - \sin^2 2x}$

Multiplying both numerator by $\sec^2 2x$

$\Rightarrow \int \frac{4\sec^2 2x\,dx}{4\sec^2 2x - \sin^2 2x \sec^2 2x} = \int \frac{4\sec^2 2x}{4(1+\tan^2 2x) - \tan^2 2x} dx = \int \frac{4\sec^2 2x}{4 + 3\tan^2 2x} dx$

Sub. $\sqrt{3}\tan 2x = t \Rightarrow 2\sqrt{3}\sec^2 2x\,dx = dt$

$$\Rightarrow \int \frac{\frac{2}{\sqrt{3}}dt}{4+t^2} = \frac{2}{\sqrt{3}} \times \frac{1}{2} \tan^{-1}(t/2) + c = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}\tan 2x}{2}\right) + c$$

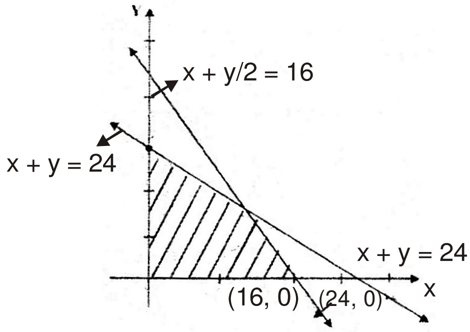
40. Let x no. of rings and y no. of chains are is manufactured

$\Rightarrow x + y <= 24$ ... (1) (quantity constraint)

Time constraint $x \times 1 + y + \frac{1}{2} <= 16$ ... (2) and $x \geq 0,\ y \geq 0$

Corner points are (0, 0), (16, 0), (0, 32) and (8, 16)

Profit = 300x + 190 y



P (0, 0) = 0

P (16, 0) = 4800

P (0, 32) = 5820

P (8, 16) = 5440

Profit is maximum when he makes only chains.

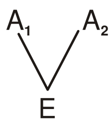
41. Let A_1 is event lost card is club

A_2 is event lost card is not club

E from given pack two cards are drawn and both are club.

By Bayes theorem

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1)\,P\left(\frac{E}{A_1}\right)}{P(A_1)\,P\left(\frac{E}{A_1}\right) + P(A_2)\,P\left(\frac{E}{A_2}\right)}$$



$$P(A_1) = \frac{13}{52},\ P(A_2) = \frac{39}{52} \quad \Rightarrow P\left(\frac{E}{A_1}\right) = \frac{^{12}C_2}{^{51}C_2},\ P\left(\frac{E}{A_2}\right) = \frac{^{13}C_2}{^{51}C_2}$$

Sub. and get answer.