Syllabus: Integral In-Differential Calculus, Vectors, 3D, Quadratic Equation, Sequence and Series, Binomial Theorem

MATHS

Section I

Straight objective type

*This section contains 8 multiple-choice questions numbered 1 to 8. Each question has 4 choices (A), (B), (C) and (D), out of which **only ONE** is correct.*

1. Consider $f(x)=\begin{cases} |f(x)| & ; \ |x|\geq 1 \\ ax^2+b & ; \ |x|<1 \end{cases}$. If equation of tangent at x = -1 is x – y + 2 = 0 while equation of tangent at x = 1 is x + y –2 = 0 then

   $(a)\ a=-\frac{1}{2}\ \&\ b=\frac{3}{2}$ $\quad (b)\ a=-\frac{1}{2}\ \&\ b=-\frac{2}{3}$ $\quad (c)\ a=\frac{1}{2}\ \&\ b=\frac{3}{2}$ $\quad (d)\ a=\frac{1}{2}\ \&\ b=\frac{3}{2}$

2. If $|\cos^{-1} 1/n| < \pi/2$ then

   $\lim_{n\to\infty}\left((n+1)\frac{2}{\pi}\cos^{-1}\frac{1}{n}-n\right)$ is equal to

   $(a)\ \frac{2-\pi}{\pi}$ $\quad (b)\ \frac{\pi-2}{\pi}$ $\quad (c)\ 1$ $\quad (d)\ 0$

3. Let $f(x)=\begin{cases} \ln(a+e^{2}+1)-\sin x & ; \ x\leq 0 \\ \int_0^2 |x-t|\,dt & ; \ 0<x<2 \\ \sin^{-1}(3x^2-6x+b)+\frac{4x-\pi}{4} & ; \ x\geq 2 \end{cases}$

   then, value of 'a' and 'b' for which f(x) is continuous function at x = 0 and x = 2.

   $(a)\ a=-1,\ b=\frac{1}{\sqrt{2}}$ $\quad (b)\ a=-e^{2},\ b=0$ $\quad (c)\ a=1,\ b=-\frac{1}{\sqrt{2}}$ $\quad (d)\ a=-1,\ b=0$

4. Let a, b, c $\in$ R such that no two of them are equal and satisfy $\begin{vmatrix} 2a & b & c \\ b & c & 2a \\ c & 2a & b \end{vmatrix}=0$ then equation $24ax^2 + 4bx + c = 0$ has

   (a) at least one in [-1/2, 0] (b) at least one in [1/2, 1]
   (c) at least one in [0, 1/2] (d) exacting one in [0, 1]

5. Which of the following does not represent a straight line
   (a) ax + by + cz + d = 0, a'x + by + cz + d = 0 (a $\neq$ a')
   (b) ax + by + cz + d = 0, ax + b'y + cz + d = 0 (b $\neq$ b')
   (c) ax + by + cz + d = 0, ax + by + c'z + d = 0 (c $\neq$ c')
   (d) ax + by + cz +d = 0, ax + by + cz + d' = 0 (d $\neq$ d')

6. If $\int_0^x f(x)\sin t\,dt=\text{constant};\ x\in(0,2\pi)$ and $f(\pi)=2$, then $f\left(\frac{\pi}{2}\right)=$

   $(a)\ \frac{1}{\sqrt{2}}$ $\quad (b)\ \frac{1}{\sqrt{2}+1}$ $\quad (c)\ 2$ $\quad (d)\ 4$

7. $\int \frac{x^2\tan x}{x\tan x+1}\,dx$ is equal to –

   $(a)\ \frac{x^2}{2}+\ln|x\sin x+\cos x|+C$ $\quad (b)\ \frac{x^2}{2}-\ln|x\tan x+1|+C$
   $(c)\ \frac{x^2}{2}-\ln|x\sin x+\cos x|+C$ $\quad (d)\ \frac{x}{2}-\ln|x\sin x+\cos x|+C$

8. $\int_0^{\infty}\left(\frac{\pi}{1+\pi^2x^2}-\frac{1}{1+x^2}\right)\ln x\,dx$ is equal to

(a) $-\frac{\pi}{2}\ln\pi$ (b) 0 (c) $\frac{\pi}{2}\ln 2$ (d) None of these

Section – II

Straight Objective Type (More than one options may be correct) **(+4, 0)**

9. If $\lim_{x\to 0}\sin\left(\frac{\ln(1-\cos^m x)}{x^n}\right)$ exists, where $m, n \in N$, then

(a) $m \in N, n = 1$ (b) $m \in N, n = 2$ (c) $m \in N, n = 3$ (d) $m \in N, n = N$.

10. Identify the statement(s) which is/are incorrect?

(a) $\vec{a}\times\left[\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right] = \left(\vec{a}\times\vec{b}\right)\left|\vec{a}\right|$

(b) If $\vec{a}, \vec{b}, \vec{c}$ are non-zero, non coplanar vectors and $\vec{v}\cdot\vec{a} = \vec{v}\cdot\vec{b} = \vec{v}\cdot\vec{c} = 0$ then $\vec{v}$ must be a null vector

(c) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane containing the vectors $\vec{c}$ and $\vec{d}$ then $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right) = 0$

(d) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors then $\vec{a}\cdot\vec{b}' + \vec{b}\cdot\vec{c}' + \vec{c}\cdot\vec{a}' = 3$

11. A function y = f(x) has five collinear points (a_1, f(a_1)) where I = 1, 2, …….5 then
(a) f "(x) = 0 for all values of x. (b) f "(x) = 0 for at least three values of x.
(c) f "'(x) = 0 for all values of x. (d) f "'(x) = 0 for at least two values of x.

12. Let $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$, then $f(x) = 0$ has
(a) exactly one real root in (2,3) (b) exactly one real root in (3,4)
(c) at least one real root in (2,3) (d) None of these

Section III

This section contains 2 paragraphs C_{13-15}, and C_{16-18}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ***ONLY ONE*** *is correct.*

C_{13-15}: Paragraph for Question Nos. 13 – 15

If f(x) = Mid {g(x), h(x), p(x)} means the function which will be second in order when values of the three function at a particular x are arranged?

$$f(x) = \text{Mid}\left\{x-1, (x-3)^2, 3-\frac{(x-3)^2}{2}\right\}, x \in [1, 4]$$

13. Numerical value of difference between the LHD and RHD at the point x = 2 of f(x) in x ∈ [1, 4] will be
(a) 0 (b) 2 (c) 3 (d) 1

14. The greatest value of f(x) in [1, 4] will be
(a) 1 + √3 (b) 2 + √3 (c) 3 + √3 (d) None of these

15. Rate of change of x w.r.t f(x) at x = 3 will be
(a) 1 (b) 3/2 (c) 2 (d) – 3/2

C_{16-18}: Paragraph for Question Nos. 16 – 18

Let A, B, C be the vertices of a triangle ABC in which B is taken as origin of reference and position vectors of A and C are $\vec{a}$ and $\vec{c}$ respectively. A line AR parallel to BC is drawn from A. PR (P is mid point of AB) meets AC at Q and area of triangle ACR is 2 times area of triangle ABC.

16. Position vector of R in terms of   and   is

(a) $a+2c$ (b) $a+3c$ (c) $a+c$ (d) $a+4c$

17. Position vector of Q is

(a) $\frac{2a+3c}{5}$ (b) $\frac{3a+2c}{5}$ (c) $\frac{a+2c}{5}$ (d) none of these.

18. $\frac{PQ}{QR}$ [illegible] $\frac{AQ}{QC}$ is equal to

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Section V

Matching type: Multiple matching may be there. **(+8/ 0)**

*This section contains 2 questions. And the questions contains statements given in two columns which have to be matched. Statements (a, b, c, d) in **Column I** have to be matched with Statements (p, q, r, s) in **Column II**.*

19.

Column I	Column II
(a) The dimensions of the rectangle of perimeter 36 cm, which sweeps out the largest volume when revolved about one of its sides, are	(p) 6
(b) Let A(-1, 2) and B (2,3) be two fixed points, A point P lying on y = x such that perimeter of triangle PAB is minimum, then sum of the abscissa and ordinate of point P, is	(q) 12
(c) If x_1 & x_2 are abscissae of two points on the curve $f(x) = x - x^2$ in the interval [0, 1], then maximum value of expression $(x_1 + x_2) – (x_1^2 + x_2^2)$ is	(r) 4
(d) The number of non- zero integral values of 'a' for which the function $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$ is concave upward along the entire real line is	(s) 1/2

20. Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$

Column I	Column II
(a) If -1 < x < 1, then f(x) satisfies	(p) 0 < f(x) < 1
(b) If 1 < x < 2, then f(x) satisfies	(q) f(x) < 0
(c) If 3 < x < 5, then f(x) satisfies	(r) f(x) > 0
(d) If x > 5, then f(x) satisfies	(s) f(x) < 1