Maths Solutions
TYRP-17 / Class Test-14 / Code : 1744

Q61. $\det[(\text{adj } A^T)^T] + \det[(\text{adj } A^{-1})^{-1}]$

$= \det(\text{adj } A^T) + \frac{1}{\det(\text{adj } A^{-1})} = (\det A^T)^{3-1} + \frac{1}{(\det A^{-1})^{3-1}}$

$= (\det A)^2 + \frac{1}{(\frac{1}{\det A})^2}$

$= 2(\det A)^2 = 18$

62. $\begin{vmatrix} 1+a^2p^2+2ap & 1+b^2p^2+2bp & 1+c^2p^2+2cp \\ 1+a^2q^2+2aq & 1+b^2q^2+2bq & 1+c^2q^2+2cq \\ 1+a^2r^2+2ar & 1+b^2r^2+2br & 1+c^2r^2+2cr \end{vmatrix}$

$= \begin{vmatrix} 1 & p^2 & 2p \\ 1 & q^2 & 2q \\ 1 & r^2 & 2r \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a & b & c \end{vmatrix} = 2 \begin{vmatrix} 1 & p^2 & p \\ 1 & q^2 & q \\ 1 & r^2 & r \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a & b & c \end{vmatrix}$

$= 2 \times$ (2 area of first triangle)(2 area of second triangle)

$= 2 \times 4 \times \frac{1}{2} \times 4 = 16$

63. $|A| = 6 = \det(\text{adj } A) = 6^2 = (0 - k) + 2(0+1) + 4(4k+1)$

64 Value of determinant $= -2(3\cos\theta + 4\sin\theta) - (\tan\theta + \sec^2\theta)(0)$

$+ 3(4\sin\theta + 3\cos\theta)$

$= 3(\cos\theta + 4\sin\theta = 5[\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta]$

Let $3/5 = \sin\phi = \frac{4}{5} = \cos\phi$ so $\phi \in (0, \pi/2)$

$= 5[\sin\phi\cos\theta + \cos\phi\sin\theta)$

$= 5[\sin(\phi+\theta)$

$\Rightarrow$ There is at least one $\theta \in (0, \pi/2)$ so that

$\theta + \phi = \pi/2$

Hence maximum value of determinat $= 5$

(65) Differentiating y with respect to x row wise

$$\frac{dy}{dx} = \begin{vmatrix} \cos 2 & -\sin x & \cos x - \sin x \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix} + 0 + 0$$

$$\frac{d^2y}{dx^2} = \begin{vmatrix} -\sin x & -\cos x & -\sin x - \cos x \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix} \quad \left(\begin{array}{c}\text{In both determinant} \\ \text{two rows are same}\end{array}\right)$$

$= 6$

66 $$\cot\left[\sum_{n=1}^{19} \cot^{-1}\left(1 + \frac{2n(n+1)}{2}\right)\right] = \cot\left[\sum_{n=1}^{19} \tan^{-1}\frac{1}{1+n(n+1)}\right]$$

$$= \cot\left[\sum_{n=1}^{19} \tan^{-1}\frac{n+1-n}{1+(n+1)n}\right] = \cot\left[\sum_{n=1}^{19}\left(\tan^{-1}(n+1) - \tan^{-1} n\right)\right]$$

$$= \cot\left[\tan^{-1} 20 - \tan^{-1} 1\right] = \cot \tan^{-1}\left(\frac{19}{21}\right)$$

67 Let $\cos^{-1} 2/3 = \theta \Rightarrow \cos\theta = \frac{2}{3}$

$$\Rightarrow \frac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2} = \frac{2}{3} \Rightarrow \tan\theta/2 = \pm\frac{1}{\sqrt{5}}$$

$$\tan\left(\frac{1}{2}\cos^{-1} 2/3\right) = \tan\theta/2$$

$\Rightarrow \tan\theta/2$ is +ive

(68) $$\tan^{-1}\left[\frac{\frac{yz}{xr} + \frac{xz}{yr}}{1 - \frac{yz}{xr}\cdot\frac{xz}{yr}}\right] = \pi/2 - \tan^{-1}\phi$$

$$\tan^{-1}\frac{\frac{z}{xyz}(y^2+x^2)}{1 - \frac{z^2}{y^2}} = \pi/2 - \tan^{-1}\phi$$

$\Rightarrow \tan^{-1}\left[\frac{z}{xyr}\cdot r^2\left(\frac{y^2+x^2}{r^2-z^2}\right)\right] = \pi/2 - \tan^{-1}\phi$

$\Rightarrow \tan^{-1}\frac{zr}{xy} = \pi/2 - \tan^{-1}\phi$

$\Rightarrow \tan^{-1}\phi = \pi/2 - \tan^{-1}\frac{zr}{xy}$

$\Rightarrow \phi = \tan\left(\pi/2 - \tan^{-1}\frac{zr}{xy}\right) = \cot\tan^{-1}\frac{zr}{xy}$

$= \frac{xy}{zr}$

(69) Taking abc common from second coloum

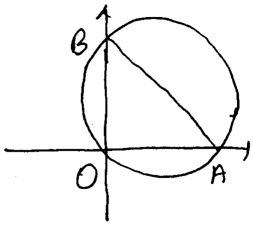
$= abc\begin{vmatrix} 1/a & 1/a & a^3 \\ 1/b & 1/a & b^3 \\ 1/c & 1/a & c^3 \end{vmatrix} = 0$

(70) Center $= (h, k) = (-2\cos\theta, \sin\theta)$

$\Rightarrow h = -2\cos\theta, \quad k = \sin\theta$

$\Rightarrow \left(\frac{h}{2}\right)^2 + k^2 = 1 \Rightarrow \frac{x^2}{4} + \frac{y^2}{1} = 1$

(71)



let centriod $= (h, k)$

$\Rightarrow A = (3h, 0)$ & $B = (0, 3k)$

$AB = 2a$

$\Rightarrow \sqrt{9h^2+9k^2} = 2a \Rightarrow 9(x^2+y^2) = 4a^2$

(72) $f(x) = \tan^{-1}\left[\sqrt{\frac{1+\cos(\pi/2 - x)}{1-\cos(\pi/2-x)}}\right] = \tan^{-1}\sqrt{\cot^2(\pi/4 - x/2)}$

$= \tan^{-1}\cot(\pi/4 - x/2) = \tan^{-1}\tan(\pi/2 - \pi/4 + x/2)$

$= \tan^{-1}\tan(\pi/4 + x/2) = \pi/4 + x/2$

(Because $x/2 \in (0, \pi/4)$ so $\pi/4 + x/2 \in (\pi/4, \pi/2)$)

so $y = f(x)$ is $y = \pi/4 + x/2$ is straight line

Normal to line $\Rightarrow$ perpendicular to above line

$\pi/ \quad \ldots \quad y - \pi/ \quad$ & slope $= \frac{-1}{k} = -2$

Hence Normal
$y - \pi/3 = -2(x - \pi/6) \Rightarrow y = -2x + 2\pi/3$
passes through $(0, 2\pi/3)$

(73) $\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0 \Rightarrow 1(\lambda+1) - \lambda(-\lambda^2+1) - 1(\lambda+1)$
$\Rightarrow \lambda^3 - \lambda = 0 \Rightarrow \lambda = 0, \pm 1$

(74) $A(\text{adj} A) = A A^T$
$|A| I = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$
$(10a + 3b) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 25a^2 - 3b & -5ab - 2b \\ 15a + 6 & -3b + 4 \end{bmatrix}$
$\Rightarrow \cancel{15a + 6 = 0} \Rightarrow 10a + 3b = 25a^2 - 3b$
$\cancel{10a + 3b}\ 0 = -5ab - 2b, \quad 15a + 6 = 0 \ \& \ 10a + 3b = -3b + 4$
$15a + 6 = 0 \Rightarrow a = -\frac{6}{15} = -\frac{2}{5}$

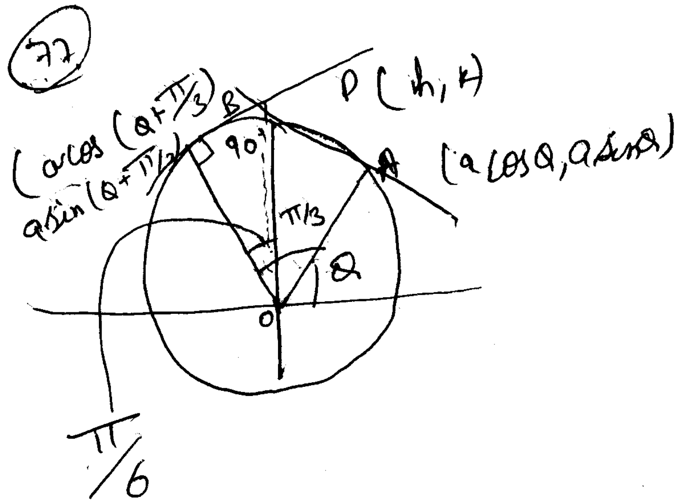
(74) $|A| I = A A^T$
$(10a + 3b) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix} = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

$\Rightarrow 15a - 2b = 0, \quad 10a + 3b = 13 \ \& \ 5a^2 + b^2 = 10a + 3b$
$\Rightarrow \begin{matrix} 15a - 2b = 0 \\ 10a + 3b = 13 \end{matrix} \Rightarrow 65a = 26 \Rightarrow a = \frac{2}{5}$
$\& \ b = 3$

(75) $\alpha + \beta + \gamma = -a, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 0, \quad \alpha\beta\gamma = -b$
Given determinant $= 3\alpha\beta\gamma - \alpha^3 - \beta^3 - \gamma^3$
$= -(\alpha + \beta + \gamma)[\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha]$
$= -(\alpha + \beta + \gamma)[(\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)]$

(76) Three lines are consistent $\Rightarrow$ lines are concurrent
$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0 = a(10-9) + 1(5-6) + 1(3-4) = 0$

(77)



In Triangle OBD

$\Rightarrow \cos \pi/6 = \dfrac{OB}{OP}$

$= \dfrac{\sqrt{3}}{2} = \dfrac{a}{\sqrt{h^2+k^2}}$

$= h^2 + k^2 = \dfrac{4a^2}{3}$

$= x^2 + y^2 = \dfrac{4a^2}{3}$

(78)

$$B = \begin{vmatrix} 1 & yz & y \\ 1 & zx & z \\ 1 & xy & x \end{vmatrix} + \begin{vmatrix} 1 & yz & z \\ 1 & zx & x \\ 1 & xy & y \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & 1/x & y \\ 1 & 1/y & z \\ 1 & 1/z & x \end{vmatrix} + xyz \begin{vmatrix} 1 & 1/x & z \\ 1 & 1/y & x \\ 1 & 1/z & y \end{vmatrix}$$

$$= \begin{vmatrix} x & 1 & xy \\ y & 1 & yz \\ z & 1 & xy \end{vmatrix} + \begin{vmatrix} x & 1 & xz \\ y & 1 & xy \\ z & 1 & yz \end{vmatrix}$$

$$\Rightarrow xyz \begin{vmatrix} x & 1 \\ y & 1 \\ z & 1 \end{vmatrix}$$

(78) $A = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ $\quad R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ $\Rightarrow \begin{vmatrix} 1 & x & y \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$

$$= (y-x)(z-x)\begin{vmatrix} 1 & x & y \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} = (y-x)(z-x)(z+x-y-x)$$

$$= (y-x)(z-x)(z-y)$$

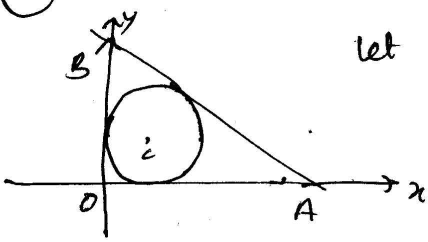
In B also $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$B = \begin{vmatrix} 1 & yz & y+z \\ 0 & zx-yz & x-y \\ 0 & xy-yz & x-z \end{vmatrix} = (x-y)(x-z)\begin{vmatrix} 1 & yz & y+z \\ 0 & z & 1 \\ 0 & x & 1 \end{vmatrix}$$

$$= (x-y)(x-z)(z-x)$$

so $A = B$

(79) Center $(2,2)$ & rad $= 2$ so circle touches the axis



let mid point of $AB = (h,k)$

$\Rightarrow A = (2h, 0)$, $B = (0, 2k)$

Eqn. of AB $\dfrac{x}{2h} + \dfrac{y}{2k} = 1$

$\Rightarrow kx + hy - 2hk = 0$ — (1)

is also tangent to circle

$\Rightarrow p = r$

$\Rightarrow \dfrac{|2h + 2k - 2hk|}{\sqrt{h^2+k^2}} = 2$ — (2)

$\Rightarrow$ ~~$|2x+2y$~~ $|2h + 2k - 2hk| = 2\sqrt{h^2+k^2}$

For line (1) $(0,0)$ & $(2,2)$ lies so same side

~~for~~ at $(0,0)$ expression on L.H.S. is $+$ive so

at $(2,2)$ it should be $-$ive

$\Rightarrow 2k + 2h - 2hk < 0$

(2) can be written as $-(2h + 2k - 2hk) = 2\sqrt{h^2+k^2}$

(80) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

$= -\left[a^3 + b^3 + c^3 - 3abc\right]$

$= -(a+b+c)\, \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

$= -\text{ve}$

(81) $\Delta = 0 = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{vmatrix} = 0 \Rightarrow a = 8$

If $a = 8$ then we can say

$2x_1 + 5x_2 + ax_3 = (x_1 + 2x_2 + 3x_3) + (x_1 + 3x_2 + 5x_3)$

$\Rightarrow b = 6 + 9$

$\Rightarrow b = 15$ for consistent

(82) $\Delta = 0$ for trivial sol. $\Rightarrow \begin{vmatrix} 1 & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & \sin\alpha & \cos\alpha \\ 0 & \cos\alpha - \sin\alpha & \sin\alpha - \cos\alpha \\ 0 & -2\sin\alpha & 0 \end{vmatrix} = 0$

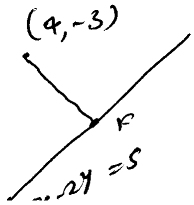
$\Rightarrow +2\sin\alpha(\sin\alpha - \cos\alpha) = 0 \Rightarrow \alpha = \pi/4$

for $\alpha \in (0, \pi/2)$

Hence both are correct & statement 2 is correct explanation.

(83) Tangent to $x^2 + y^2 = 5$ at $(1,-2)$ is $x - 2y = 5$. It is also tangent to second circle. Its point of contact with second circle is foot from center $(4,-3)$

Let foot be $\left(h, \frac{h-5}{2}\right)$



$\Rightarrow \left(\dfrac{\frac{h-5}{2} + 3}{h-4}\right) \times \frac{1}{2} = -1$

$h+1 = -4(h-4)$

(84) ~~$R_1 \to R_1 + R_2 + R_3$~~ $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+2x+(a^2+b^2+c^2)x & (1+b^2)x & (1+c^2)x \\ 1+2x+(a^2+b^2+c^2)x & 1+b^2x & (1+c^2)x \\ 1+2x+(a^2+b^2+c^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = (1-x)^2$$

so polynomial of degree 2

(85) By symmetry B & C has equal number of element
Since value of determinant can be 2, 0, -2 so A is not $B \cup C$.

(86) $$\frac{1}{(p-a)(q-b)(r-c)} \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ \frac{a}{p-a} & \frac{q}{q-b} & \frac{c}{r-c} \\ \frac{a}{p-a} & \frac{b}{q-b} & \frac{r}{r-c} \end{vmatrix} = 0$$

Since $\frac{a}{p-a} = \frac{a-p+p}{p-a} = -1 + \frac{p}{p-a}$ & others

$$\begin{vmatrix} \frac{p}{p-a} & -1+\frac{q}{q-b} & -1+\frac{r}{r-c} \\ -1+\frac{p}{p-a} & \frac{q}{q-b} & -1+\frac{r}{r-c} \\ -1+\frac{p}{p-a} & -1+\frac{q}{q-b} & \frac{r}{r-c} \end{vmatrix} = 0.$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \left(\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2\right) \begin{vmatrix} 1 & -1+\frac{q}{q-b} & -1+\frac{r}{r-c} \\ 1 & \frac{q}{q-b} & -1+\frac{r}{r-c} \end{vmatrix} = 0$$

value of determinant is 1

so $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2 = 0$

(87) $AU_1 + AU_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$A(u_1+u_2) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$u_1+u_2 = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(88) $P^3 = Q^3$ — (1)

& $P^2Q = Q^2P$ — (2)

(1) $\cancel{+}$ (2) $\Rightarrow P^3 \cancel{+} P^2Q = Q^3 \cancel{+} Q^2P$

$\Rightarrow P^2(P \cancel{+} Q) = Q^2(Q-P)$

$\Rightarrow (P^2+Q^2)(P-Q) = 0$

$\Rightarrow \det[(P^2+Q^2)(P-Q)] = 0$

$\Rightarrow \det(P^2+Q^2) \cdot \det(P-Q) = 0$

But $P \neq Q \Rightarrow P-Q \neq 0$

$\Rightarrow \det(P-Q) \neq 0$

so from above $\det(P^2+Q^2) = 0$

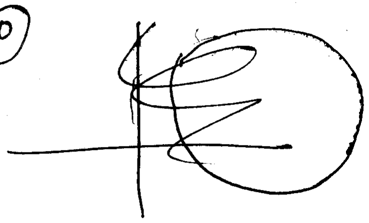
(89) $\text{adj}(\text{adj } A) = |A|^{n-2} \cdot A$

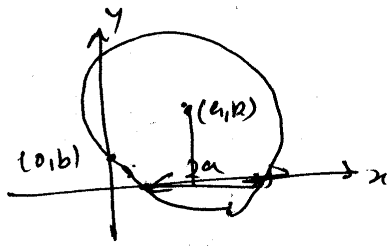
But $n = 2$ so $\text{adj}(\text{adj } A) = A$

$|\text{adj } A| = |A|^{n-1} = |A|$

So both are correct but not correct explanation

(90)





let center (h,k)

rad. $= \sqrt{h^2+(k-b)^2}$

$\perp$ distance from (h,k) to x axis $= |k|$

so $r^2 = k^2+a^2$

$\Rightarrow h^2+(k-b)^2 = k^2+a^2$

$\Rightarrow h^2+k^2+b^2-2kb = k^2+a^2$