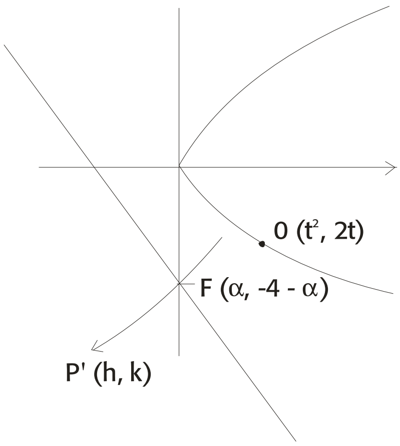
JEE (ADVANCE) SOLUTIONS – 2015 – CODE '8'
MATHEMATICS
PAPER-1

41. Let p be on curve $y^2 = 4x$ and it's image P' and F be foot from P to line
P to line $m_{PF}.m_{line} = -1$

$$\left(\frac{2t+\alpha+\alpha}{t^2-\alpha}\right)(-1) = -1$$

$\Rightarrow \quad 2t+\alpha+4 = t^2-\alpha$

$\Rightarrow \quad 2\alpha = t^2-2t-4$



So F is $\left(\frac{t^2-2t-4}{2}, -4-\frac{(t^2-2t-4)}{2}\right)$

$$= \left(\frac{t^2-2t-4}{2}, -\frac{t^2+2t-4}{2}\right)$$

$\frac{h+t^2}{2} = \frac{t^2-2t-4}{2} \Rightarrow h = -2t-4 \qquad ....(1)$

$\frac{2t+k}{-2} = \frac{-t^2+2t-4}{-2} \Rightarrow k = -t^2-4 \qquad ....(2)$

From (1) and (2)

$$k = -\left[-\left(\frac{h+4}{2}\right)\right]^2 - 4$$

$$k = -\frac{(h+4)^2}{4} - 4$$

$\Rightarrow \quad 4^y = -(x+4)^2 - 16$

$\Rightarrow \quad 4^y + 16 = -(x+4)^2$ is mirror image of curve intersection with y = – 5

$\Rightarrow \quad -20+16 = -(x+5)^2$

$\Rightarrow \quad (x+5)^2 = 4 \Rightarrow x+5 = \pm 2$

$\Rightarrow \quad x = -3, -7$

So A is (–3, –5) and B (–7, –5)

Distance bt AB = 4 units

42. Let coin is tossed n times

P (at least two head) $= 1-\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right)$

$$1-\left(\frac{1}{2}\right)^n - n\left(\frac{1}{2}\right)^n \geq 0.96$$

$\Rightarrow \quad 0.04 \geq \frac{(h+1)}{2^n}$

$\Rightarrow \quad \frac{1}{25} \geq \frac{nH}{2^n} \Rightarrow 2^n \geq 25\,(n+1)$

$\Rightarrow n \geq 8$

$\Rightarrow$ least value of n is 8

43. When girl are consecutive treat girl as one unit girls can be formed in one unit by 51
Now arrange this unit and 5 boys by = 61
So n = 5! × 6!
For in make 4 unit of girls as 1 by $5_{C_4} \times 4!$
Now arrange the boys by 5!
$\_\, B_1 \,\_\, B_2 \,\_\, B_3 \,\_\, B_4 \,\_\, B_5 \,\_$
In between their gaps in any two gabs arrange the one girl and unit = $6_{C_2} \times 2!$
So $m = 5! \times 5_{C_4} \times 4! \times 6_{C_2} \times 2!$

$$\frac{m}{n} = \frac{5! \times 5_{C_4} \times 4! \times 6_{C_2} \times 2!}{5!\,6!} = \frac{5! \times 5 \times 4! \times \frac{6\times 5}{2} \times 2}{5! \times 6!}$$

$$= 5$$

44. Latus rector = (1, 2), (1, –2)
Eqn. of tangent at (1, 2) = 24 $= \frac{4}{2}(x+1) \Rightarrow y = x+1$
Slope of normal = – 1
Eqn. of normal y – 2 = – 1 (x – 1) $\Rightarrow$ y = – x + 3
$\Rightarrow$ x + y – 3 = 0
Is tangent to circle
$\Rightarrow$ b = r

$$\frac{(3-2-3)}{\sqrt{1+1}} = r \Rightarrow r = \sqrt{2} \Rightarrow r^2 = 2$$

45. $f(x) = \begin{cases} -1 &, -1 \leq x < 0 \\ 0 &, 0 \leq x < 1 \\ 1 &, 1 \leq x < 2 \\ 0 &, x \geq 2 \end{cases}$

$$I = \int_{-1}^{0} \frac{x\,f(x^2)}{2+f(x+1)}dx + \int_{0}^{1} \frac{x\,f(x^2)}{2+f(x+1)}dx + \int_{1}^{\sqrt{2}} \frac{x\,f(x^2)}{2+f(x+1)}dx + \int_{\sqrt{2}}^{2} \frac{x-f(x^2)}{2+f(x+1)}dx$$

$$= \int_{-1}^{0} 2\times 0 \; dx + \int_{0}^{1} 0\, dx + \int_{1}^{\sqrt{2}} \frac{x\times 1}{2+0}dx + \int 0 \; dx$$

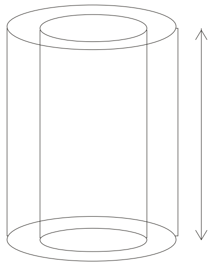
$$= \int_{1}^{\sqrt{2}} \frac{x}{2}dx = \frac{1}{4}x^2\Big|_{1}^{\sqrt{2}} = \frac{1}{\alpha}[2-1] = \frac{1}{4}$$

So $\quad 4I - 1 = 0$

46. $V_{solid} = \left(\pi r_0^2 h - \pi r_i^2 h\right) + \pi r_0^2 \times 2$

$r_0 = r_i + 2$

$V = \pi r_i^2 h$



$V_{solid} = \pi h\left(r_0^2 - r_i^2\right) + \pi (r_i + 2)^2 \times 2$

$= \pi (r_0 - r_i)(r_0 + r_i) \times \frac{V}{\pi r_i^2} + 2\pi(r_i + 2)^2$

$= \frac{\pi \times 2(r_i + 2 + r_i) \times V}{\pi r_i^2} + 2\pi(r_i + 2)^2$

$= \frac{2V(2r_i + 2) \times V}{r_i^2} + 2\pi(r_i + 2)^2$

$\frac{dv}{dr_i} = 4b\left(-\frac{1}{r_i 2} - \frac{2}{r_i 3}\right) + 4\pi (r_i + 2)$

$\frac{dv}{dr_i} = 0$ when $r_i = 10$

$\Rightarrow 4V\left(-\frac{1}{100} - \frac{2}{1000}\right) + 4\pi\ 812$

$\Rightarrow 4V\left(\frac{10+2}{1000}\right) = 12 \times 4\pi \Rightarrow \frac{4 v \times 12}{1000} = 12 \times 4\pi$

$\Rightarrow V = 1000\pi$ so $\frac{V}{250\pi} = 4$

47. $F'(a) + 2 = \int_b^a f(x)\, dx$

$2\left[\cos^2\left(a^2 + \frac{\pi}{6}\right)\right] \times 2a - 2\cos^2 a = \int_0^a f(x)\, dx$

Diff. w.r.t. to a

$4 \cos^2\left(a^2 + \frac{\pi}{6}\right) + 4a \times 2 \cos\left(a^2 + \frac{\pi}{6}\right)\sin\left(a^2\ f\frac{\pi}{6}\right)(-2a) = f(a) + 4 \sin a \cos a$

Sub. $a = 0$

$\Rightarrow f(0) = 4\left(\cos^2 \frac{\pi}{6} + 0\right)$

$= 4\left(\frac{\sqrt{3}}{2}\right|^2 = 3$

48. $\frac{5}{4}\cos^2 2x + \left(\cos^4 x + \sin^4 x\right) + \left(\cos^6 x + \sin^6 x\right) = 2$

$\frac{5}{4}\cos^2 2x + 1 - 2\sin^2 x \cos^2 x + 1 - 3\sin^2 x \cos^2 x = 2$

$\Rightarrow \quad \frac{5}{4}\cos^2 2x - \frac{1}{2}\sin^2 2x - \frac{3}{x}\sin^2 2x = 0$

$\Rightarrow \quad \frac{5}{4}\cos^2 2x = \frac{5}{4}\sin^2 2x \Rightarrow \tan^2 2x \Rightarrow 2x \in \ [0, 4\pi]$

$[0, 4\pi]$ is four period and every period there are 2 sol. so total no of sol. = 8

49. $\left(1+e^x\right)\frac{dy}{dx} \times ye^x = 1 \qquad \Rightarrow \quad \frac{d}{dx}\left[\left(1+e^x\right)4\right] = 1$

On integrating $(1+e^x)\,4 = x + c$

$x = 0 \ \Rightarrow \ 4 = 2$

$\Rightarrow \ (1+1)\times 2 = 0 + c \Rightarrow c = 4$

So $\quad y\ (1+e^x) = x+4$

$y = (x+4)\ \ (1+e^x)$

So (a) $y(-4) = 0$

$\frac{dy}{dx} = (x+4)\,e^x + 1 + e^x$

$= e^x(x+5) + 1$

For critical points $e^x(x+5)+1 = 0$

Let $h(x) = e^x(x+5)+1$

$h^1(x) = e^x\ (x+5) + e^n$

$= e^x\ (x+6) = 0 \quad \frac{h^1(x)\ldots\ldots\ldots +++}{-6}$

$\Rightarrow$ x = 6 is point of minima of h(x)

So h min = h (–6) = $e^{-6}(-6+5)+1 > 0$

$\Rightarrow$ minimum value of h(x) is $>$ 0 so $h(x) \neq 0$

$\Rightarrow$ y(x) does not posses critical point

50. Let center be (α, α) and radius = r

Eqn. of circle $\ (x-\alpha)^2 + (y-\alpha)^2 = r^2 \quad \ldots(1)$

Diff. w.r.t. to x

$\Rightarrow \ 2\,(x-\alpha) + 2(y-\alpha)\frac{dy}{dx} = 0$

$\Rightarrow \ (x-\alpha) + (y-\alpha)\frac{dy}{dx} = 0 \qquad \ldots(2)$

Again diff. w.r.t. to x

$1 + \left(\frac{dy}{dx}\right)^2 + (y-\alpha)\frac{d^2y}{dx^2} = 0 \qquad \ldots.(3)$

From (2)

$x + 4\frac{dy}{dx} = \left(1+\frac{dy}{dx}\right)\alpha \qquad \ldots(4)$

Sub. α from (4) in (3)

$$1 + \left(\frac{dy}{dx}\right)^2 + \left[y - \left(\frac{x + y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)\right]\frac{d^2y}{dx^2} = 0$$

$$1+\left(\frac{dy}{dx}\right)^2+\frac{(y-x)\frac{d^2y}{dx^2}}{1+\frac{dy}{dx}}=0$$

$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]\left[1+\frac{dy}{dx}\right]+(y-x)\frac{d^2y}{dx^2}=0$$

$$\Rightarrow (y-x)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+\left(\frac{dy}{dx}\right)^2+\frac{dy}{dx}+1=0$$

$$\Rightarrow (y-x)\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)\left[\left(\frac{dy}{dx}\right)^2+\frac{dy}{dx}+1\right]+1=0$$

$\Rightarrow P=y-x,\ Q=\left(y^1\right)^2+y^1+1$ so (b) and (c) are correct.

51. $\lim_{x\to 0^-}\frac{x}{|x|}g(x)=-1g(0)=0$

$\lim_{x\to 0^+}\frac{x}{|x|}g(x)=1g(0)=0\Rightarrow f(x)$ is continuous

$$f(x)=\begin{cases}-g(x) & , \ x<0\\ 0 & , \ x=0\\ g(x) & , \ x>0\end{cases}\qquad \begin{aligned}f'(0^-)&=-g'(x)\Big|_{x=0}=-g'(0)=0\\ f'(0^+)&=+g'(x)\Big|_{x=0}=g'(0)=0\end{aligned}$$

$\Rightarrow$ t is differentiable at $x=0$

$$h(x)=\begin{cases}e^{-x} & , \ x\le 0\\ e^{x} & , \ x\ge 0\end{cases}\quad h(x) \text{ is cont. at } x=0$$

$$\begin{aligned}h'(0^-)&=e^{-x}\Big|_{x=0}=-1\\ h'(0^+)&=e^{x}\Big|_{x=0}=1\end{aligned}\quad\Rightarrow h(x) \text{ is not diff. at } x=0$$

$$foh=t\{(x)\}=f(e^{|x|})=\frac{e^{(x)}}{\left|e^{|x|}\right|}g(e^{|x|}),\ \forall x\in R$$

$$=g(e^{|x|}),\ x>0$$

$$=\begin{cases}g(e^{x}) & , \ x>0\\ g(e^{-x}) & , \ x\le 0\end{cases}$$

foh is cont.

$foh\,(0^-)=g'(e^x)\,e^x/_{x=0}=g'(1)$

$foh\,(0^+)=g'(e^x)\left[e^{-x}\right]/_{x=0}=-g'(x)$

$\Rightarrow$ foh is not diff. at $x=0$

$$ho\,f=h\,\{f(x)\}=\begin{cases}h\left(\frac{x}{|x|}g(x)\right) & , \ x\ne 0\\ h\,(0) & , \ x=0\end{cases}$$

$$=\begin{cases}e^{\left|\frac{x}{|x|}g(x)\right|} & , \ x\ne 0\\ 1 & , \ x=0\end{cases}$$

$$= \begin{cases} e^{|g(x)|} & , \quad x \neq 0 \\ 1 & , \quad x = 0 \end{cases}$$

If g(x) > 0 then hot is diff. at x = 0 if g(x) < 0 $\text{hot} = \begin{cases} e^{-g(x)} & , \quad x < 0 \\ 1 & , \quad x = 0 \\ e^{g(x)} & , \quad x > 0 \end{cases}$

Hof $(0^-) = e^{-g(x)}\{-g'(x)\}\big|_{x=0} = 0$ and similarly hof $(0^+) = 0$

52. $f(x) = \sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right]$

$-1 \le \sin x \le 1 = 1 - \frac{\pi}{2} \le \frac{\pi}{2}\sin y \le \frac{\pi}{2}$

$-1 \le \sin\left(\frac{\pi}{2}\sin x\right) \le x$

$-\frac{\pi}{6} \le \left(\frac{\pi}{6}\sin\frac{\pi}{2}\sin x\right) \le \frac{\pi}{6}$

$-\frac{1}{2} \le \sin\left(\frac{\pi}{6}\sin\frac{\pi}{2}\sin x\right) \le \frac{1}{2}$

fog $= f\,\{\frac{\pi}{2}\sin x) = \sin\left[\frac{\pi}{6}\sin\left\{\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right\}\right]$

range of fog = is also $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$\frac{\pi}{6}\left[\sin\left(\frac{\pi}{2}\sin x\right)\right]$

gof = g {f(x)} $= \frac{\pi}{2}\sin\left[\sin\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right]$

since $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

so gof = $\frac{\pi}{2}\sin\{f(x)\} \in \left[-\frac{\pi}{2}\sin\frac{1}{2}, \frac{\pi}{2}\sin\frac{1}{2}\right]$

$\text{gof}_{max.} = \frac{\pi}{2}\left(\sin\frac{1}{2}\right)$

Since $\frac{\pi}{6} > \frac{1}{2}$

$\Rightarrow \sin\frac{\pi}{6} > \sin\frac{1}{2}$

$\Rightarrow \frac{1}{2} > \sin\frac{1}{2}$

So $\sin\frac{1}{2} < \frac{1}{2}$

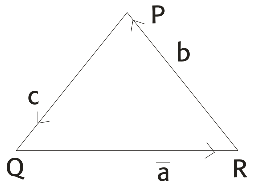
$\Rightarrow \text{gof}_{max.} = \frac{\pi}{2}\sin\frac{1}{2}$

$< \frac{\pi}{2}.\frac{1}{2} = \frac{\pi}{\alpha} < 1$

So (D) is incorrect.

53. $\bar{a}+\bar{b}+c=0$

$\bar{b}+\bar{c}=-\bar{a}$



Taking mod

$|\bar{b}+\bar{c}|=|-a|$

$\Rightarrow \quad |b|^2+|c|^2+2\bar{b}.\bar{c}=|a|^2$

$\Rightarrow \quad 48+|c|^2+2\times 24=14.4$

$\Rightarrow \quad |c|^2=48$

(A) $\frac{|c|^2}{2}-|a| = 24 - 12 = 12$ is correct

(B) $\frac{|c|^2}{2}-|a|=24+12=36$ is incorrect

(C) $|\bar{a}\times\bar{b}+\bar{c}\times\bar{a}|=|\bar{a}\times(\bar{b}-\bar{c})|$

$=|-(\bar{b}+\bar{c})\times(\bar{b}-\bar{c})|$

$=|(\bar{b}+\bar{c})\times(\bar{c}-\bar{b})|$

$=|\bar{b}\times\bar{c}-0+0-\bar{c}\times\bar{b}|$

$=2|\bar{b}\times\bar{c}|$

$=2|b||c|\sin\theta$

$=2(4\sqrt{3})\ (4\sqrt{3})\frac{\sqrt{3}}{2}$

$=48\sqrt{3}$

$\bar{b}\,.\,\bar{c}=24$

$4\sqrt{3}\times 4\sqrt{3}\ \cos\theta=24\ 6$

$\cos\theta=\frac{1}{2}$

$\Rightarrow \theta=\frac{\pi}{3}$

$\bar{a}+\bar{b}=-\bar{c}$

$|\bar{a}+\bar{b}|^2=|c|^2$

$\Rightarrow |a|^2+|b|^2+2\bar{a}.\bar{b}=|c|^2$

$\Rightarrow \quad 144+48+2\,\bar{a}\,.\,\bar{b}=48$

$\bar{a}.\bar{b}=-72$

54. (A) $\left(Y^3Z^4-Z^4Y^3\right)^T=\left(Z^T\right)^4\left(Y^T\right)^3-\left(Y^T\right)^3\left(Z^T\right)^4$

$=Z^4(-Y)^3-(-Y)^3Z^4$

$=-Z^4Y^3+Y^3Z^4$

$= Y^3Z^4 - Z^4Y^3 \Rightarrow$ symmetric

(B) $\left[\left(X^{44}+Y^{44}\right)\right]^T = \left(X^T\right)^{44} + \left(Y^T\right)^{44}$

$= (-X)^{44} + (-Y)^{44}$

$= X^{44} + Y^{44} \Rightarrow$ symetric

(C) $\left(X^4Z^3 - Z^3X^4\right)^T = \left(Z^T\right)^3\left(X^T\right)^4 - \left(X^T\right)^4\left(Z^T\right)^3$

$= Z^3(-X)^4 - (-X)^4(Z)^3$

$= Z^3X^4 - X^4Z^3$

$= -(X^4Z^3 - Z^3X^4)$

$\Rightarrow$ skew symmetric

(D) Similarly D is also skew symmetric

55. $R_3 \to R_3R_2$ and $R_2 \to R_2 - R_1$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ (5+2\alpha) & 5+4\alpha & 5+6\alpha \end{vmatrix} = -648\alpha$$

$C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$

$$\begin{vmatrix} (1+\alpha)^2 & \alpha(3\alpha+2) & \alpha(2+5\alpha) \\ 3+2\alpha & 2\alpha & 2\alpha \\ 5+2\alpha & 2\alpha & 2\alpha \end{vmatrix} = -648\alpha$$

$$= \alpha^2\begin{vmatrix} (1+\alpha)^2 & 3\alpha+2 & 2+5\alpha \\ 3+2\alpha & 2 & 2 \\ 5+2\alpha & 2 & 2 \end{vmatrix} = -648\alpha$$

$C_3 \to C_3 - C_2$

$$\Rightarrow \alpha^2\begin{vmatrix} (1+\alpha)^2 & 3\alpha+2 & 2\alpha \\ 3+2\alpha & 2 & 0 \\ 5+2\alpha & 2 & 0 \end{vmatrix} = -648\alpha$$

$\Rightarrow \alpha^2[2\alpha(6+4\alpha-10-4\alpha)] = -648\alpha$

$\alpha^2(-8\alpha) = -648\alpha \Rightarrow \alpha^3 = 81\alpha \Rightarrow \alpha = 01 \pm 9$

56. Eqn. of P_3 is $x+2-1+\lambda y = 0$

Distance of this plane from (0, 1, 0) $= \dfrac{|0+0-1+\lambda|}{\sqrt{1+1+\lambda 2}} = 1$

$\Rightarrow |\lambda - 1| = \sqrt{2+\lambda^2}$

$\Rightarrow \lambda^2 + 1 - 2\lambda = 2 + \lambda^2$

$\Rightarrow 2\lambda = -1 \Rightarrow \lambda = -\dfrac{1}{2}$

So plane is $x + z - 1 - \dfrac{1}{2}y = 0 \Rightarrow 2x - y + 2z - 2 = 0$

$\dfrac{|2\alpha - \beta + 2\gamma - 2|}{\sqrt{4+1+4}} = 2 \Rightarrow |2\alpha - \beta + 2\gamma - 2| = 6$

$\Rightarrow\ 2\alpha-\beta+2\gamma-2=\pm 6$

$\Rightarrow\ 2\alpha-\beta+2\gamma=8$ ...(1)

And $2\alpha-\beta+2\gamma=-4$ ...(2)

57. Since line is constant distance from both the planes

$\Rightarrow$ line is || to both planes

$\Rightarrow$ line is $\perp n_1=\hat{i}+2\hat{j}-\hat{k}$

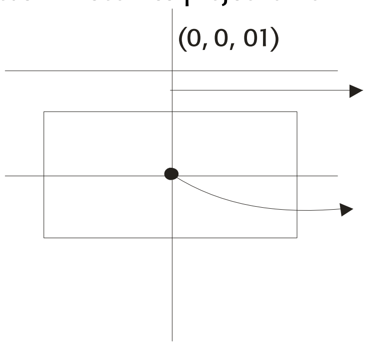
$\perp n_2=2\hat{i}-\hat{j}+\hat{k}$

$\Rightarrow$ line is || to $\bar{n}_1\times\bar{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ 2- & -1 & 1\end{vmatrix}$

$=\hat{i}-3\hat{j}-5\hat{k}$

So line is $\frac{x}{1}=\frac{y}{-3}=\frac{z}{-5}$

Locus M becomes projection of line on the plane.



$\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}=\lambda$

$F=(\lambda, 2\lambda, -\lambda)$ lies on the plane

$\Rightarrow\ \lambda+4\lambda+\lambda+1=0$

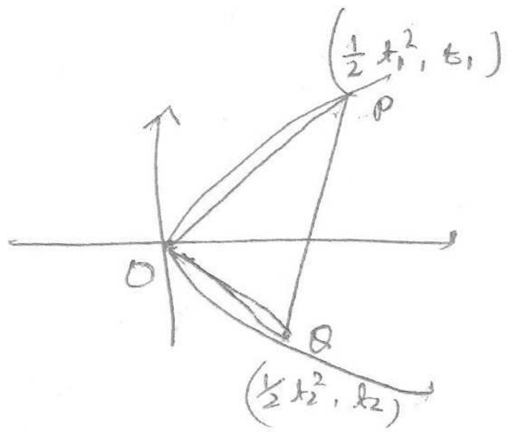
$\Rightarrow\ \lambda=-\frac{1}{6}$ so F is $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$

So Eqn. of locus is

$\frac{x-\left(-\frac{1}{6}\right)}{1}=\frac{4-\left(-\frac{1}{3}\right)}{-3}=\frac{z-\frac{1}{6}}{-5}$

A and B lies on the line

58. PQ is diameter of circle passing through O



$\Rightarrow \ <POQ = 90 \qquad \Rightarrow \ M_{OP}.M_{OQ} = -1$

$\dfrac{t_1}{\frac{1}{2}t_1^2} \times \dfrac{t_2}{\frac{1}{2}t_2^2} = -1 \Rightarrow \dfrac{4}{t_1t_2} = -1 \quad \Rightarrow t_1t_2 = -4 \qquad \ldots(1)$

$\text{Area } = \dfrac{1}{2}OP.OQ$

$3\sqrt{2} = \dfrac{1}{2}\sqrt{\dfrac{1}{4}t_1^4 + t_1^2} \quad \sqrt{\dfrac{1}{4}t_2^4 + t_2^2}$

$6\sqrt{2} = |t_1||t_2|\sqrt{\dfrac{1}{4}t_1^2 + 1} \quad \sqrt{\dfrac{1}{4}t_2^2 + 1}$

$6\sqrt{2} = |-4|\sqrt{\left(\dfrac{1}{4}t_1^2 + 1\right)} \quad \sqrt{\dfrac{1}{4} \times \dfrac{16}{t_1^2} + 1}$

$\dfrac{3\cancel{6}\sqrt{2}}{\cancel{4}\,2} = \sqrt{\left(\dfrac{1}{4}t_1^2 + 1\right)\sqrt{\left(\dfrac{4}{t_1^2} + 1\right)}}$

$\Rightarrow \dfrac{9}{2} = \left(\dfrac{1}{\alpha}.t_1^2 + 1\right)\left(\dfrac{4}{t_1^2} + 1\right) = 1 + 1 + \dfrac{1}{4}t_1^2 + \dfrac{4}{t_1^2} \quad \Rightarrow \dfrac{5}{2} = \dfrac{t_1^2}{4} + \dfrac{4}{t_1^2} \quad \Rightarrow \dfrac{5}{2} = \dfrac{t_1^4 + 16}{4 + t_1^2} \Rightarrow t_1^4 + 16 = 10\, t_1^2$

$t_1^4 - 10\, t_1^2 + 16 = 0$

$t_1^2 = 8, 2$

$\Rightarrow t_1 = 2\sqrt{2}, \sqrt{2}$

$P = \left(\dfrac{1}{2}t_1^2, t_1\right) = \left(4_1^2\sqrt{2}\right) \ \& \ \left(1, \sqrt{2}\right)$

60. (A) $2\,(a^2 - b^2) = c^2$

$2\,(\sin^2 x - \sin^2 y) = \sin^2 z$

$\Rightarrow \ 2\sin(x+y)\sin(x-y) = \sin^2 z$

$\Rightarrow \ 2\sin(\pi - z)\sin(x-y) = \sin^2 z$

$\Rightarrow \ \dfrac{\sin(x-y)}{\sin z} = \dfrac{1}{2} = \lambda$

$\cos\left(n\pi \times \dfrac{1}{2}\right) = 0 \Rightarrow \cos\left(n\dfrac{\pi}{2}\right) = 0 \quad \Rightarrow \ n = 1, 3, 5 \ \Rightarrow \ P, R, S$

(B) $1+\cos 2x-2\cos^2 y = 2\sin x \ \sin y$

$1+1-2\sin^2 x-2+4\sin^2 y = 2\sin x \sin y$

$\Rightarrow \ 4\sin^2 y-2\sin^2 x = 2\sin x \sin y$

$\Rightarrow \ 2\sin^2 y-\sin^2 x = \sin x \sin y$

$\Rightarrow \ (2\,R\,b)^2-(2\,Ra)^2 = (2\,Ra)\ (2R\,b)$

$\Rightarrow \ 2b^2-a^2 = ab$

$\Rightarrow \ 2b^2-ab-a^2 = 0$

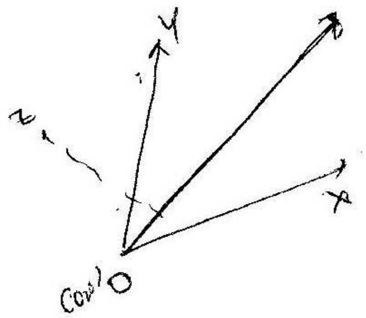
$2b^2-2ab+4b-a^2 = 0$

$2b\,(b-a)+a\,(b-a) = 0$

$(2b+a)\,(b-a) = 0$

$\Rightarrow \ b = a \Rightarrow \frac{a}{b} = 1$

(C) Bisector of OX and OY $\left(\frac{\sqrt{b}\,i+j}{2}+\frac{i+\sqrt{3}\,\hat{j}}{2}\right)$ vector is along the $\left(\frac{\sqrt{3}+1}{2}\right)(\hat{i}+\hat{j})$



Vector $\hat{i}+\hat{j}$ maker angle of 45° with time x axis so eqn. of bisector $y-x=0$

distance from $(\beta, 1-\beta) \ = \frac{|1-\beta-\beta|}{\sqrt{1+1}} = \frac{3}{\sqrt{2}}$

$\Rightarrow \ |1-2\beta| = 3$

$\Rightarrow \ 2\beta-1 = \pm 3$

$\Rightarrow \ \beta = 2, -1$

So P and Q

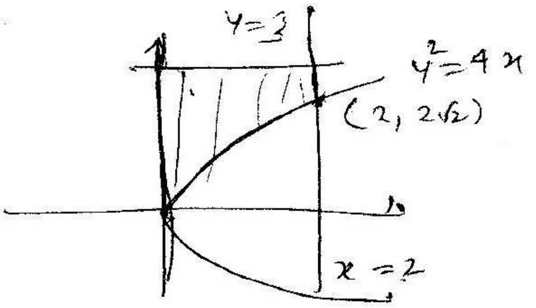
(D) $y = |\,d_{x-1}\,|+|\,d_{x-2}\,|+ax$

When $\alpha = 0 \ \Rightarrow y = 3$

$F(0) = 3\times 2-\int_0^2 2\sqrt{x}\,dx$

$= 6-\frac{2\times 2\times 2^{3/2}}{3}$

$= 6-\frac{4}{3}\times 2\sqrt{2} = 6-\frac{8}{3}\sqrt{2}$



When $\alpha = 1$

$y = |x-1| + |x-2| + x$