Permutation and Combination

Q1 $\underbrace{{}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3}$

$\Rightarrow {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + \underbrace{{}^{48}C_3 + {}^{48}C_4} = {}^{51}C_3 + {}^{50}C_3 + \underbrace{{}^{49}C_3 + {}^{49}C_4}$

$= {}^{52}C_4 = {}^{52}C_{48}$

Q2 Number of ways two vertices can be joined $= {}^{8}C_2$ = no. of sides + no of diagonal

$\Rightarrow {}^{8}C_2 = 8 +$ no. of diagonals.

$\Rightarrow$ no. of diagonal $= {}^{8}C_2 - 8$

Q3 $n!\,(21-n)! = \dfrac{n!\,(21-n)!}{21!} \times 21! = \dfrac{21!}{{}^{21}C_n}$ is minimum

when ${}^{21}C_n$ is maximum ie $n = 10$ or 11

Q4 No. of pairs of two circles $= {}^{8}C_2$

Every pair gives two intersection so maximum point of intersection $= {}^{8}C_2 \times 2$

Q5 Three type of intersection

(i) Circle - Circle

${}^{4}C_2 \times 2$

(ii) circle - line

${}^{4}C_1 \times {}^{4}C_1 \times 2$

(iii) line - line.

${}^{4}C_2 \times 1$

So total is sum of (i), (ii) (iii)

Q6 The hall is can illuminate by switching on at least on light

$= 2^{10} - 1$

Q7 Total no of ways $= (7-1)!$

No. of ways when two persons are together $= 2! \times (6-1)!$
$= 2!\,5!$

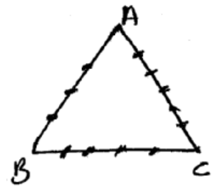
Hence required ways $= 6! - 2!\,5! = 480$

Q8 Select three grades & arrange then on students $= {}^{4}P_3$

9. Six identical coin 3 of them have 3 of them have tail ⇒ 3 identical objects of one type & 3 identical objects of other type are to be arranged in a row so no. of ways = $\frac{6!}{3!3!}$

10. $\_ - \_ - \_ - \_$ ⟶ these places are to be filled by 1,1, 3,3 & remaining by 2,2,4

so no. of numbers $= \frac{4!}{2!2!} \times \frac{3!}{2!}$

11. 

Total no. of points $= 12$

No. of ways three points can be selected $= {}^{12}C_3$

= {No. of triangles if no 3 or more points are co-linear}

But 3, 4, 5 points are co-linear in AB, BC, CA lines

No. of ways of selecting 3 points on these lines are ${}^{3}C_3, {}^{4}C_3, {}^{5}C_3$ which does not form triangle hence total triangle

$= {}^{12}C_3 - {}^{3}C_3 - {}^{4}C_3 - {}^{5}C_3$

12. No. of words having at least one repeated letter = Total − when no letter ~~repeat~~ is repeated

$= 10^5 - 10\times9\times8\times7\times6$

13. A number is divisible by three ⇒ sum of digits in the number is divisible by 3

Sum of given six digits is 15 (divisible by 3)

So digit which can be left is 0 or 3

So five digits can be

0, 1, 2, 4, 5 or 1, 2, 3, 4, 5

$\underline{4}\times\underline{4}\times\underline{3}\times\underline{2}\times\underline{1}$ No. of numbers $\underline{5}\times\underline{4}\times\underline{3}\times\underline{2}\times\underline{1}$

96 120

So total no. $= 96 + 120 = \underline{\underline{216}}$

14. Total no. of hand shakes = no. of ways of selecting two persons

$= {}^{n}C_2 = 66$

$\frac{n(n-1)}{2} = 66 \Rightarrow n(n-1) = 132$

$\Rightarrow n = 12$

15 2 front. Front seat can filled by drivers (ie 2) ③
5×4 rear. then rear seat by remaining
so total $= 2\times5\times4 = 40$

16 No. of way of answering all the 3 questions $= 4\times4\times4 = 64$
out of this in only one way all the three questions are correct
so. no. of ways student fails to answer all question correctly $= 64-1 = 63$

18 11 can be obtained by following ways.

1. — 6, 4, 1	no. of ways of obtaining 6, 4, 1 in different orders $= 3! = 6$	
2 — 6, 3, 2	$\rightarrow 3! = 6$	
3 — 5, 5, 1	$\rightarrow \frac{3!}{2!} = 3$	
4 — 5, 4, 2	$\rightarrow 3! = 6$	
5 — 5, 3, 3	$\rightarrow 3!/2! = 3$	
6 — 4, 4, 3	$\rightarrow \frac{3!}{2!} = 3$	

so Total is $6+6+3+6+3+3 = 27$

19 m_1 = Total − when I & N are together.
$$= \frac{7!}{2!} - 2\times\frac{6!}{2!} = \frac{7!}{2!} - 6! = 6!\left(\frac{7}{2}-1\right) = \frac{720\times5}{2} = 1800$$

$$m_2 = \frac{5!}{2!} = 60$$

$$\frac{m_1}{m_2} = 30$$

20 $a = (x+2)!$, $b = {}^{x}P_{11} = \frac{x!}{(x-11)!}$, $c = (x-11)!$
$$a = 182\,bc$$
$$(x+2)! = 182\times\frac{x!}{(x-11)!}\times(x-11)! = 182\,x!$$
$$(x+2)(x+1)\cancel{x!} = 182\times\cancel{x!}$$
$$= 14\times13 \Rightarrow x = 12$$

21 No. of diagonals $= {}^{n}C_2 - n = 44$
$$\frac{n(n-1)}{2} - n = 44 \quad \text{find } n.$$

22. Number can be → 1 Digit → 6 = 6
→ 2 digit → 5×5 = 20
→ 3 digit → 5×5×4 = 100

Total = 131

23. No. of words coming before debac
Starting with a → 4!
" " b → 4!
" " c → 4!
word starting with d have debac
d a → 3!
d b → 3!
d c → 3!
d e a → 2!
d e b a c

So Total words before debac
= 24+24+24 + 6+6+6+2
= 92
so at 93rd place.

24. Let $A = \{a_1, a_2, a_3 - - - a_n\}$
Set P & Q can have any no. of elements of A & B
& each element of A have 4 possible ways with respect to ~~is~~ P & Q.

ie

P	✓	×	✓	×
Q	✓	×	×	✓

✓ → element is present in that particular set
× — Not present

If ~~P & Q do not~~ $P \cap Q = \phi$
⇒ no element is simultaneously present in both ~~A & B~~ P & Q
So each element have three ways so no. of ways elements can be placed in P & Q = 3·3 — — 3 up to n times
$= 3^n$

25. 4 S, 3 A, 2 I, 2 N, T, O

selection of 4 letters → (i) All identical
(ii) 3 identical + one diff.
(iii) 2 + 2 type
(iv) 2+1+1 type
(V) 1+1+1+1 type.

8

	ways of selection		
All identical	1	1	
3 + 1	$^2C_1 \times {^5C_1}$	10	
2 + 2	$^4C_2$	6	Total = 72
2 + 1 + 1	$^4C_1 \times {^5C_2}$	40	
1 + 1 + 1 + 1	$^6C_2$	15	

26. Total no. of seven digit no. $= 9 \times 10^6$

Half of them have sum even & half of them have sum odd

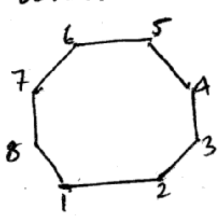
So Total no. of numbers in which sum is even $= \frac{9 \times 10^6}{2}$

27. Total no of triangles = no. of ways of selecting 3 vertices $= {^8C_3}$

No. of triangle having no side of octagon to be side of octagon = Total − one side is same − two side is same

When one side is same $= 8 \times 4$

8 → selection of side; 4 → ways of selection of third vertex not adjacent to given two vertex (selected side)



When two side are same ⇒ selection of three adjacent vertices = 8 ie [(1,2,3), (2,3,4) − − − − (8,1,2)]

Hence required $= {^8C_3} - 32 - 8 = 16$

28. The selected no. in A.P. can have ~~com~~ common difference

$1, 2, 3, 4, - - - \frac{n-1}{2}$,

common diff	Triplets	no. of ways
1	(1,2,3), (2,3,4) − − − − (n−2, n−1, n)	n−2
2	(1,3,5) (2,4,6) − − − (n−4, n−2, n)	n−4
3	(1,4,7), (2,5,8) − − (n−6, n−3, n)	n−6
⋮		⋮
$\frac{n-1}{2}$	$(1, \frac{n+1}{2}, n)$	1

So total no of ways of selecting three numbers

$$= (n-2) + (n-4) + (n-6) + \text{———} + 1$$

$$= \left(\frac{n-1}{2}\right)\left[n-2+1\right] = \frac{(n-1)^2}{2}$$

29 5 will appear in 1, 2, 3 digit numbers.

In 1 digit → 1 times

~~In 2 digit 5 5 2 times~~

In 2 digit
nonfive 5 → 8 times
5 nonfive → 9 times
5 5 → 2 times

In 3 digit.
5 5 _ → 9×2 times
_ 5 5 → 8×2 "
5 _ 5 → 9×2 "

5 _ _ → 9×9 times
_ 5 _ → 8×9 "
_ _ 5 → 8×9 "

5 5 5 → 3 "

Total no of ~~times~~ times 5 appears $= 1 + 8 + 9 + 2 + 18 + 16 + 18 + 81 + 72 + 72 + 3 = 300$

30 — — Every ball have two two possibility so
Total no of ways of distributing n balls into two boxes $= 2^n$
But there are two possibilities in which all the balls are in one box. Hence required no. of ways $= 2^n - 2$

31 Signals may contain different no. of flags

(i) 1 Flag signal → 6
(ii) 2 " " → 6×5 → $\boxed{\begin{matrix} 6 \\ 5 \end{matrix}}$
(iii) 3 " " → $6 \times 5 \times 4$
(iv) 4 " " → $6 \times 5 \times 4 \times 3$
(v) 5 " " → $6 \times 5 \times 4 \times 3 \times 2$
(vi) 6 " " → $6 \times 5 \times 4 \times 3 \times 2 \times 1$

Total is addition of all 6 cases.

32 Total no. of numbers = Total no. of 20 digit numbers that can be formed with 0, 1, 2, 3, 4 where 0 can occurs at any place & any no. of times $= 5 \times 5 \times 5$ -- upto 20 times

$= 5^{20}$

Alt. 1 Digit no. $= 5$

2 " " $= 4 \times 5$

3 " " $= 4 \times 5^2$

20 " " $= 4 \times 5^{19}$

So total $= 5 + 4 \times 5 + 4 \times 5^2 + \text{———} \; 4 \times 5^{19}$

$= 5 + \dfrac{4 \times 5(5^{19} - 1)}{5-1} = 5^{20}$

33 Ans → ${}^{n-2}C_3$ ~~(Consider n objects are~~

If $n-3$ objects are placed in a row then between them & sides their are $n-2$ ~~gap~~ gaps. In these gaps ~~no~~ 3 Objects ~~can~~ in predefined order can be placed by ${}^{n-2}C_3$ ways. Now taking it's reverse Process of n objects are arranged~~ment~~ in row then 3 objects in which no two of them are consecutive can be selected by ${}^{n-2}C_3$ way.

34 If all the four digits are in the number then two cases are possible.

(i) A digit is repeated three times & three other digits

Total no of such ~~digit~~ numbers $= {}^4C_1 \times \dfrac{6!}{3!}$ → arrangements of six number in which 3 are identical.

Selecting the repeated digit.

ii) When two digits are repeated two times & other two are diff.

~~Total~~ Total no. of such numbers $= {}^4C_2 \times \dfrac{6!}{2!2!}$

Hence the required numbers = (i) + (ii)

35

— — — — — — — — First select two places for two specified speakers.

$= {}^8C_2$

They can arraged by only 1 way

Remaining six can be arranged by $6!$ ways

Total $= {}^8C_2 \times 6!$

36 Total no. of point of intersection of given lines $= {}^nC_2$

$= \frac{n(n-1)}{2}$

Since a line is ~~met by~~ intersected by $n-1$ different line so $n-1$ points are co-linear in each of the n given line

No. of fresh line by joining their point of intersection

$$= {}^{\frac{n(n-1)}{2}}C_2 - n.\left({}^{n-1}C_2\right)$$

→ no. of ways of joining two points out of $n-1$ co-linear points on every line.

$$= \frac{\left(\frac{n(n-1)}{2}\right)\left(\frac{n(n-1)}{2}-1\right)}{2} - n \cdot \frac{(n-1)(n-2)}{2}$$

37 Number of ways when a particular child always goes

$= {}^7C_2$

(One particular child + two other children from 7 remaining children)

38 ${}^9C_5 - {}^7C_3$ = Total − two particular friends are invited together

39

$V — x — x — x — x$ Remove N & arrange remaing.

$= \frac{4!}{3!}$

Now there are five gabs in which 3 N can be arranged by ${}^5C_3$ ways so total $= \frac{4!}{3!} \times {}^5C_3$